Manifold Sampling for Nonconvex Optimization of Piecewise Linear Compositions

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Problem statement

We are interested in solving the problem:

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad f(x) \triangleq \psi(x) + h(F(x))$$

where $\psi : \mathbb{R}^n \to \mathbb{R}$, $F : \mathbb{R}^n \to \mathbb{R}^p$, $h : \mathbb{R}^p \to \mathbb{R}$,

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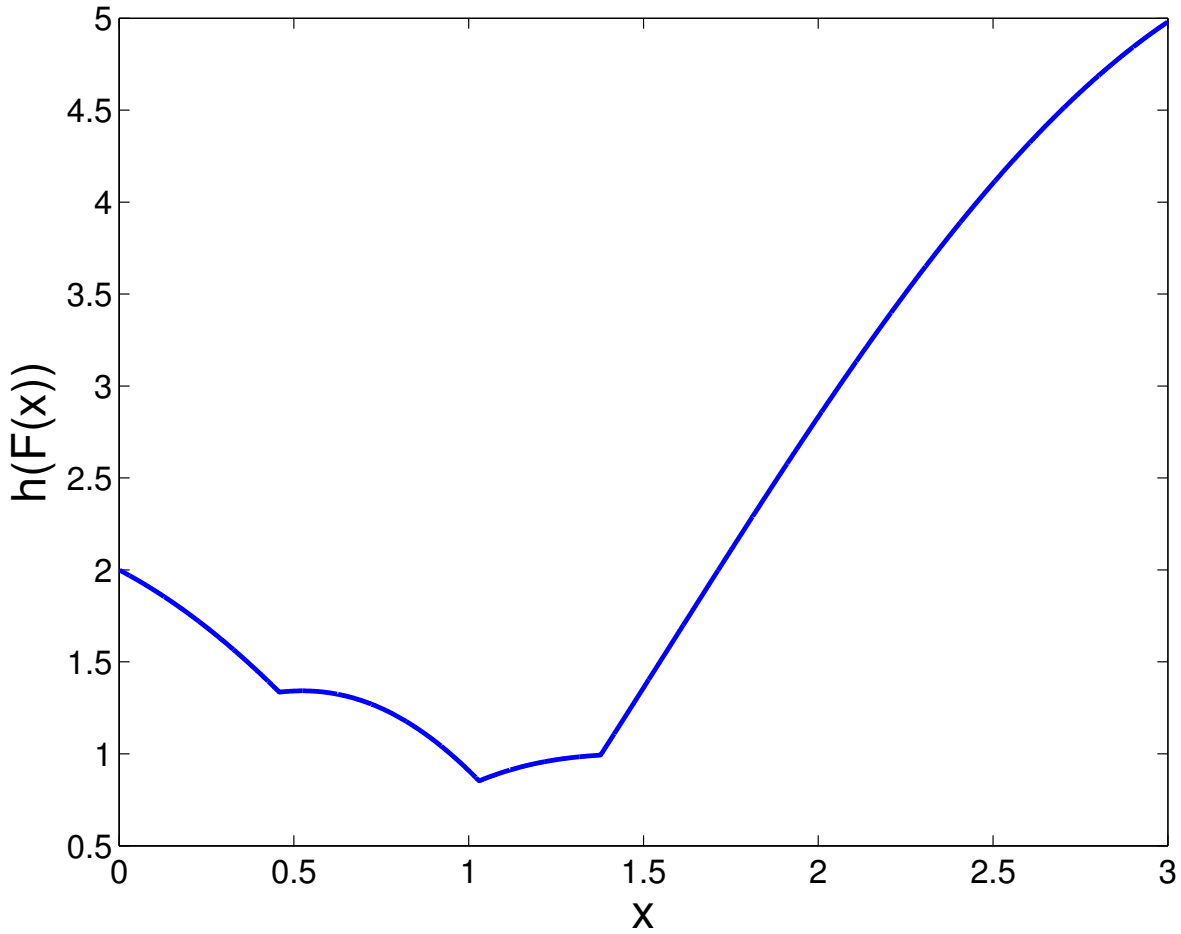
- F is smooth, nonlinear, and has a relatively unknown structure (expensive to evaluate)

Piecewise linear h does not imply $h \circ F$ is piecewise linear.

Formulation

$$h(F(x)) = \max\{\sin(2x) + 1, \cos(2x), x\} - \min\{\sin(2x) + 1, \cos(2x), x\}$$



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- We will build component models m^{F_i} of each F_i around points x. We can then use $\nabla M(x) \in \mathbb{R}^{n \times p}$ where

$$\nabla M(x) \triangleq \left[\nabla m^{F_1}(x), \ldots, \nabla m^{F_p}(x)\right].$$

Piecewise linear functions

Definition

A function $h\colon \mathbb{R}^p \to \mathbb{R}$ is *piecewise linear* if h is continuous and there exists a finite collection $\mathfrak{H} \triangleq \{h_i : i = 1, \ldots, \hat{m}\}$ of affine functions that map $\mathbb{R}^p$ into $\mathbb{R}$, for which

$$h(z) \in \left\{\tilde{h}(z) : \tilde{h} \in \mathfrak{H}\right\}, \qquad \forall z \in \mathbb{R}^p.$$

- h is a *continuous selection* of $\mathfrak{H}$.
- Elements of $\mathfrak{H}$ are *selection functions of* h.
- $h_i : z \in \mathbb{R}^p \mapsto \langle a_i, z\rangle + b_i$ for each i.

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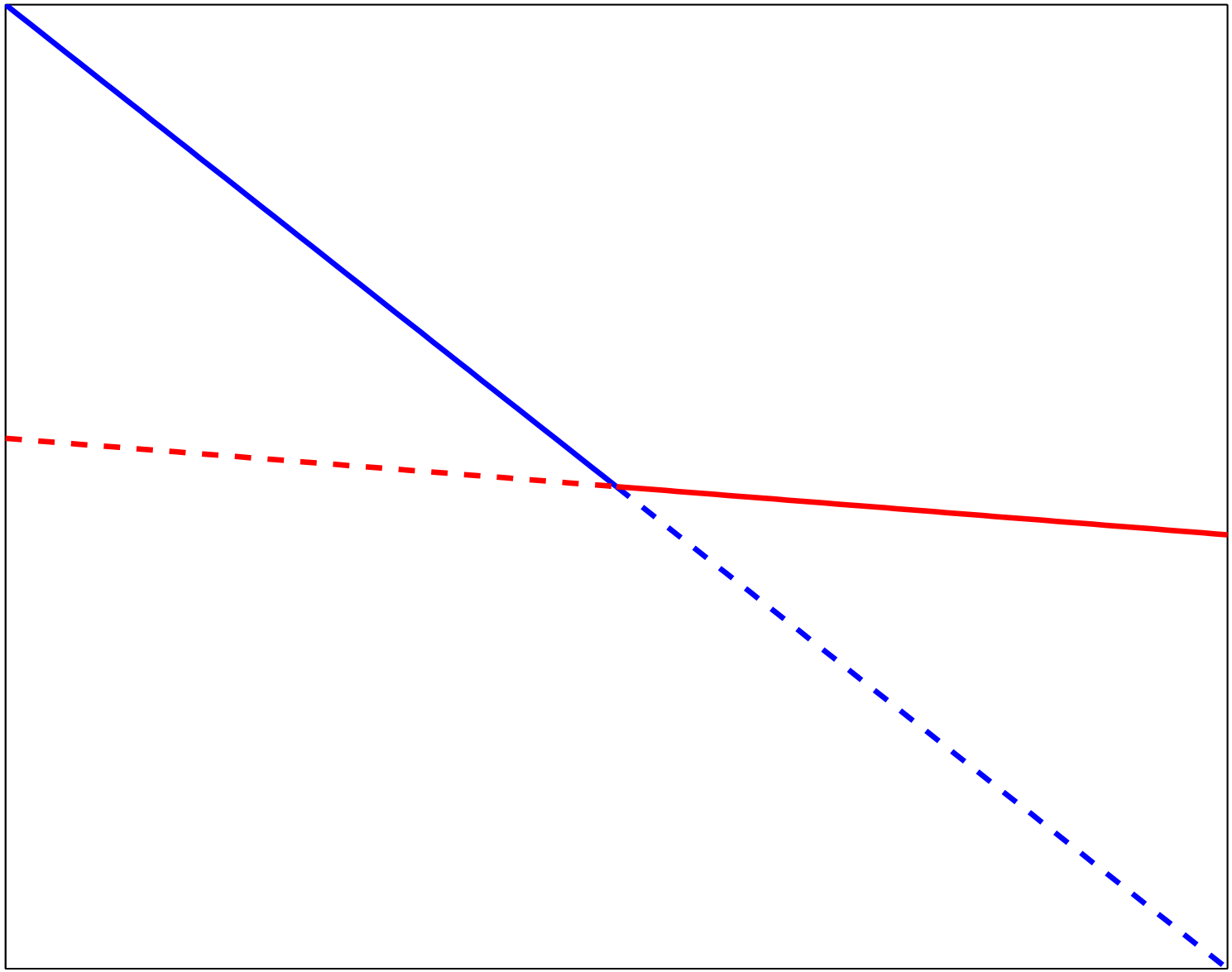
Definition

$$\mathcal{S}_i \triangleq \{y : h(y) = h_i(y)\}, \quad \tilde{\mathcal{S}}_i \triangleq \mathbf{cl}\left(\mathbf{int}\left(\mathcal{S}_i\right)\right), \quad I_h(z) \triangleq \left\{i : z \in \tilde{\mathcal{S}}_i\right\},$$

h_i for $i \in I_h(z)$ is an *essentially active selection function* for h at z.

Essentially active



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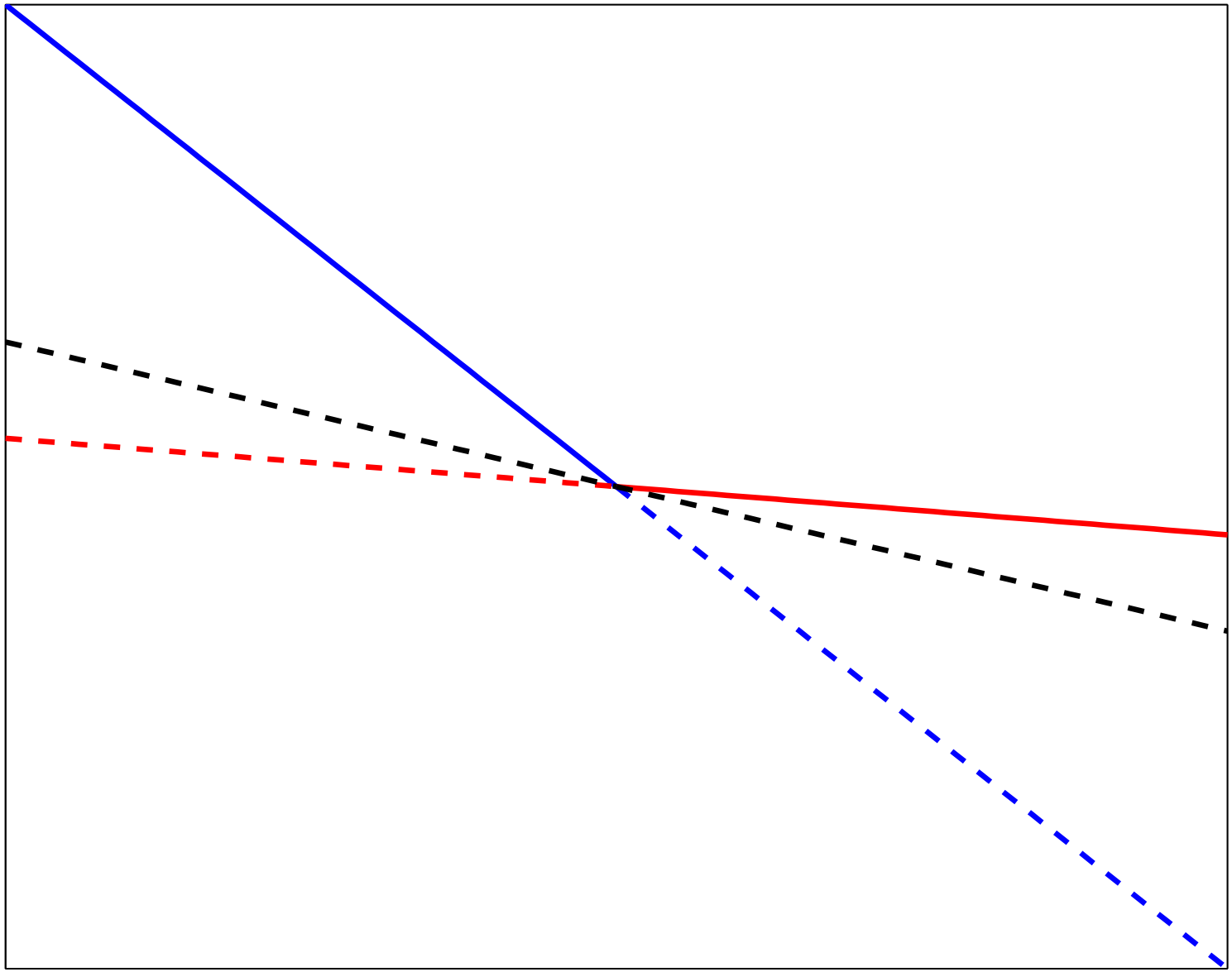
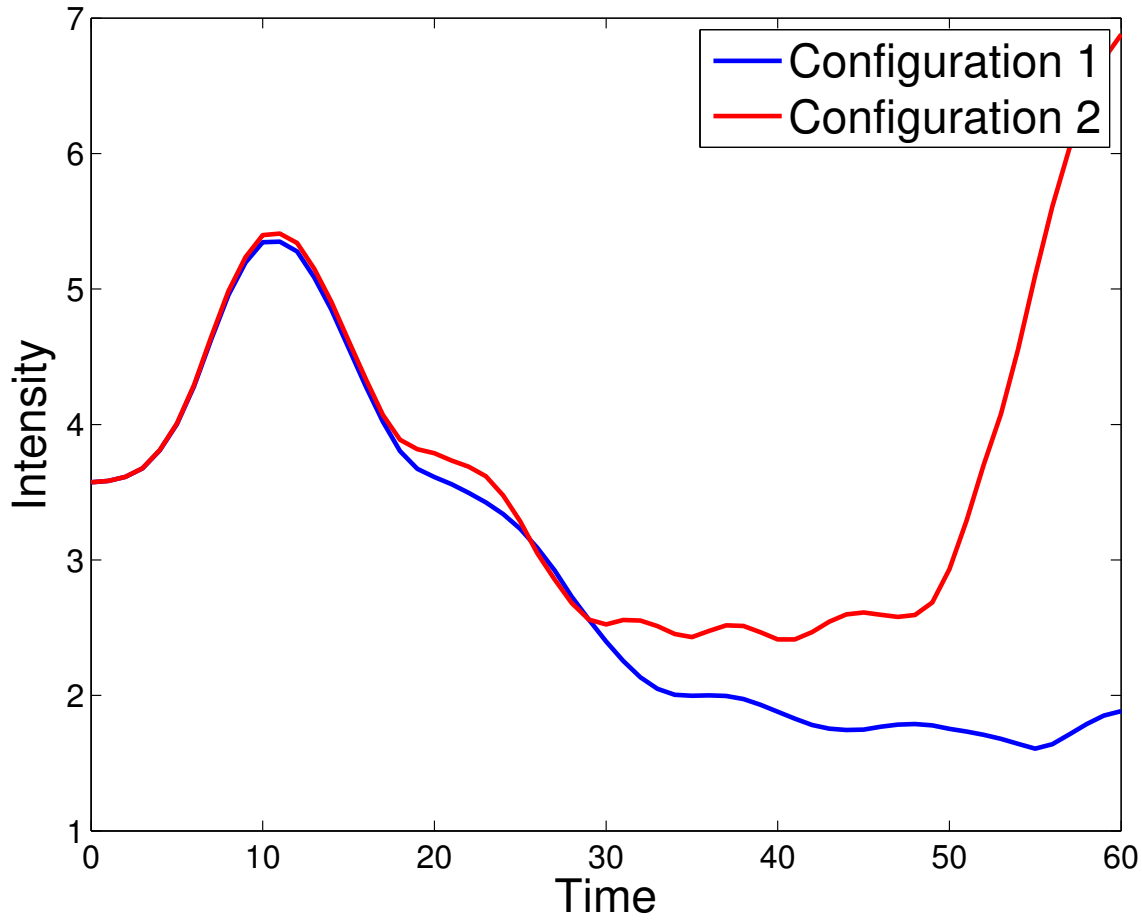


Laser pulse propagating in a plasma channel

Determine plasma channel properties that minimize the maximum difference in the laser intensity.



$$f(x) = \max_{\Omega_1} \{F_i(x)\} - \min_{\Omega_2} \{F_i(x)\}$$

A generalized derivative

Definition

The B-*subdifferential* of f at x is defined as

$$\partial_{\mathrm{B}} f(x) \triangleq \left\{ \xi : \xi = \lim_{y^j \to x} \nabla f(y^j) : \; y^j \in \mathcal{D} \right\}.$$

The *generalized Clarke subdifferential* of f at x is defined as

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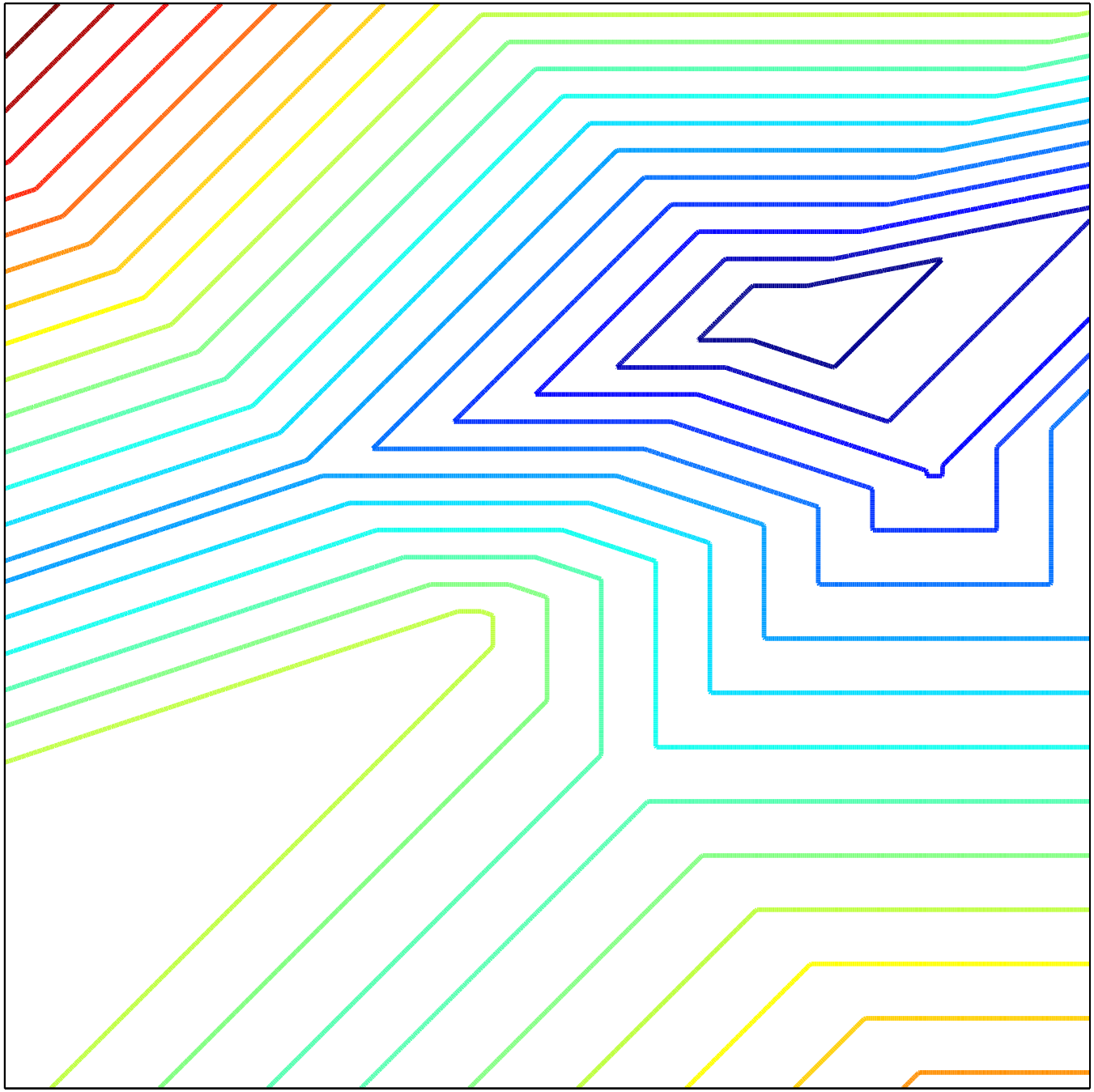
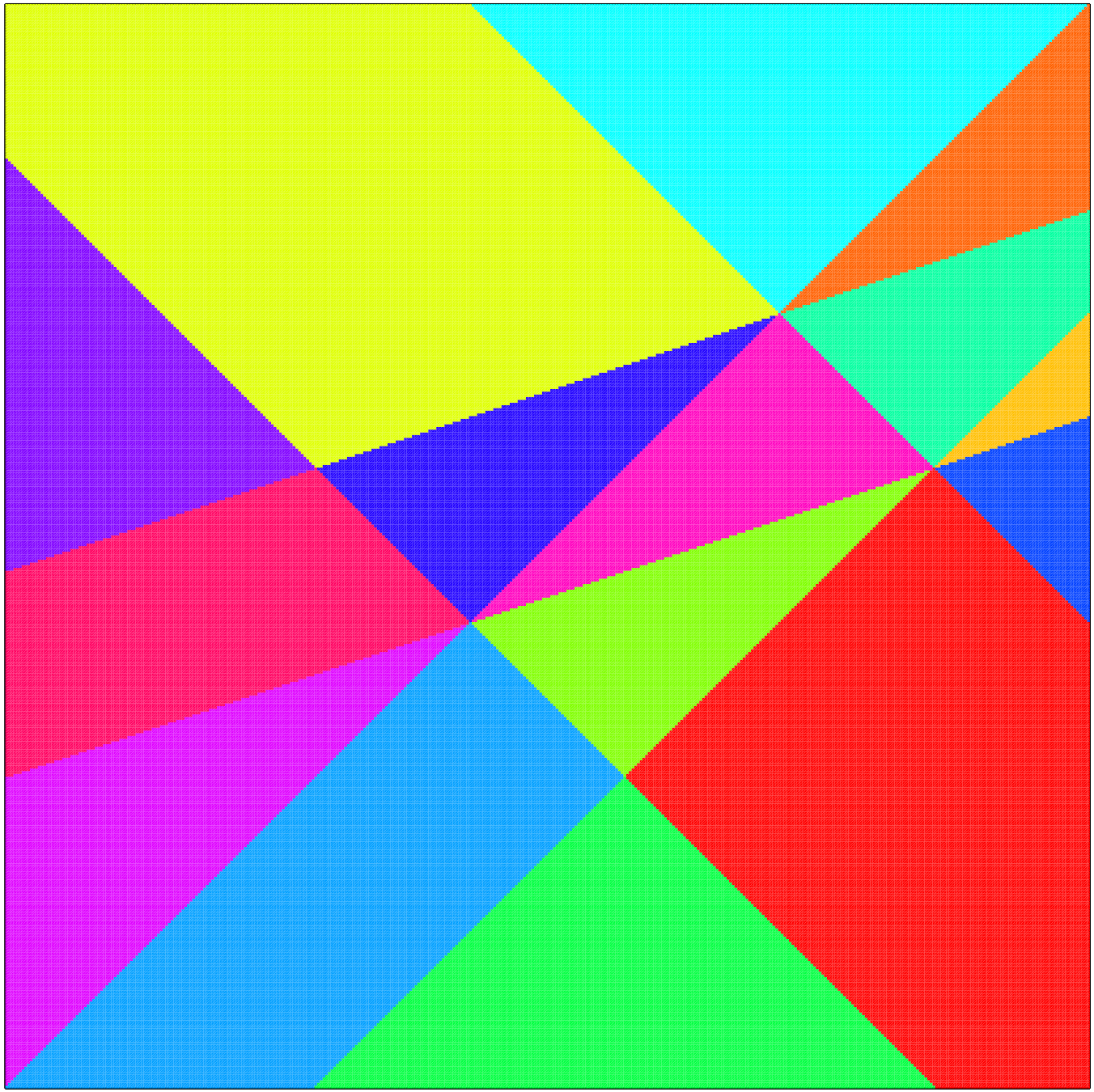
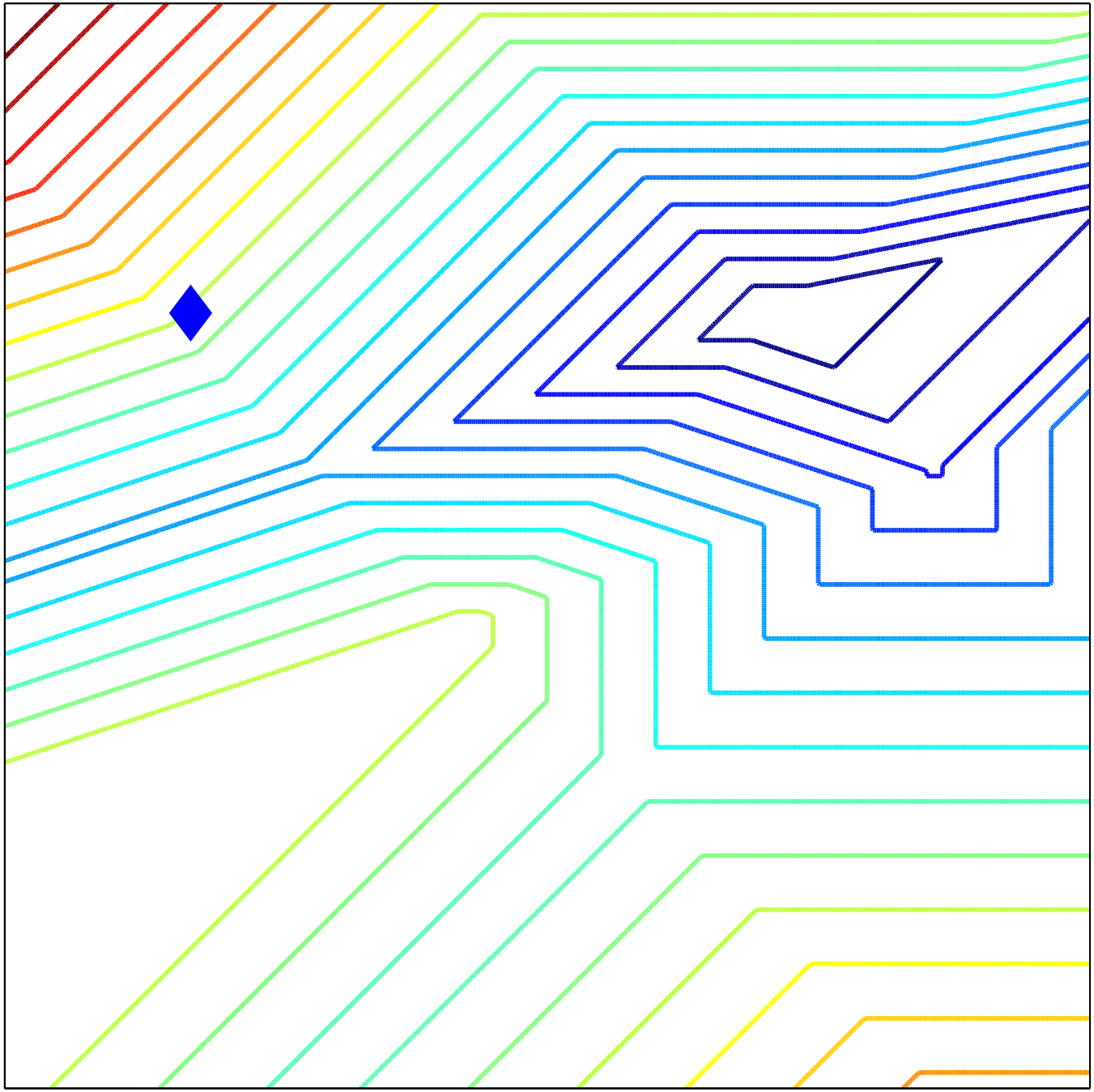
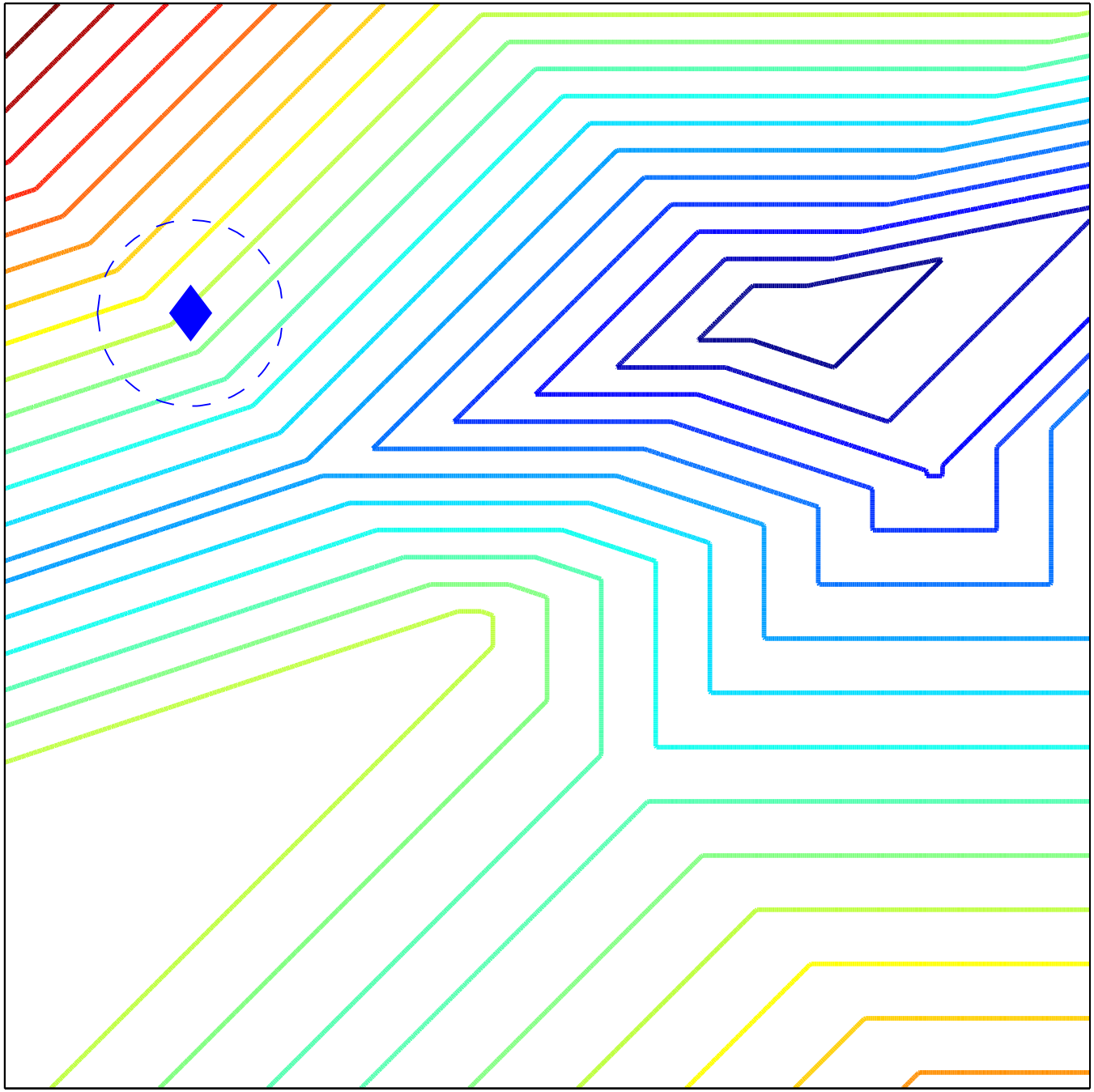
Definition

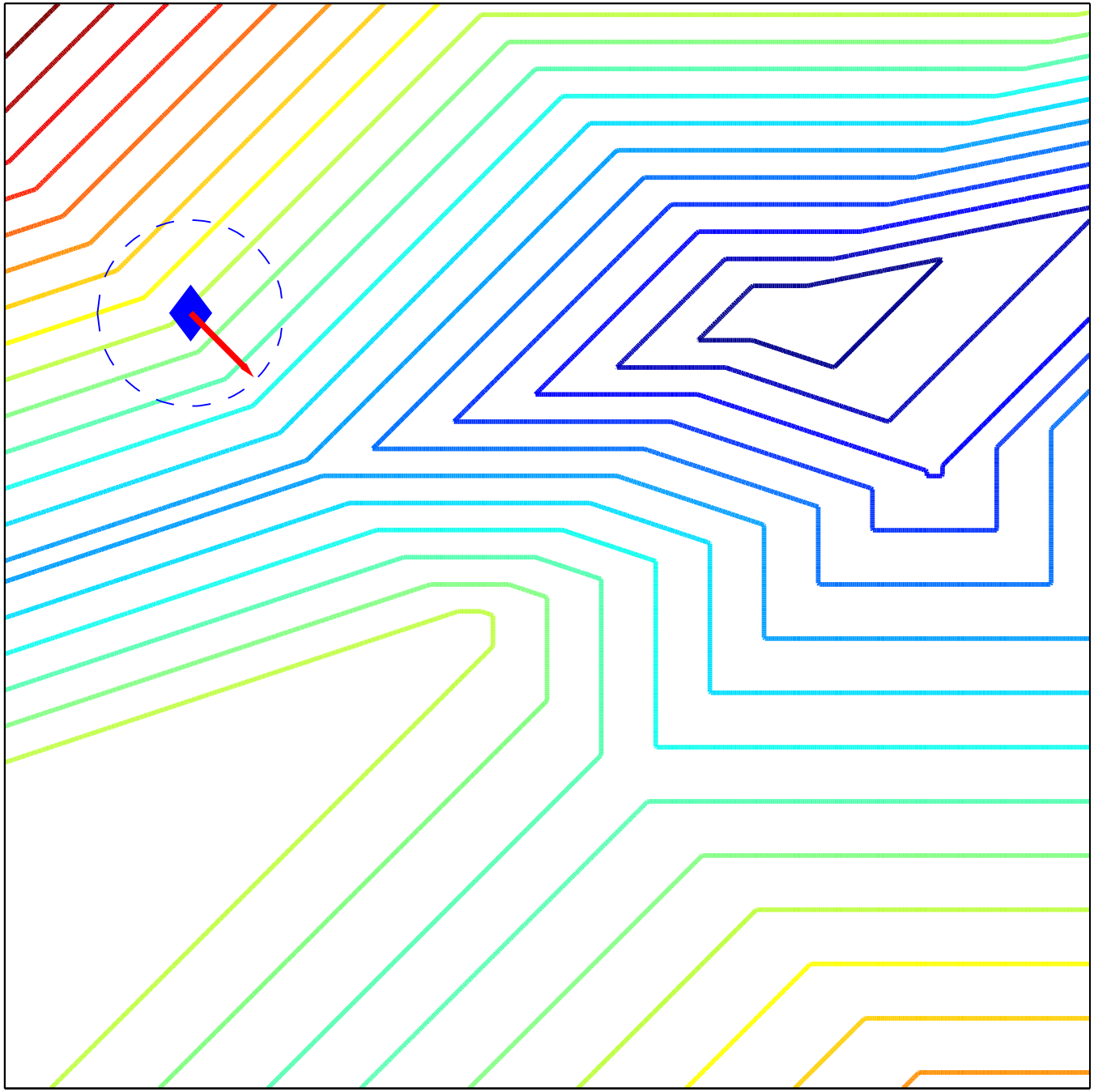
A point x is called a *Clarke stationary* point of f if $0 \in \partial_{\mathrm{C}} f(x)$.

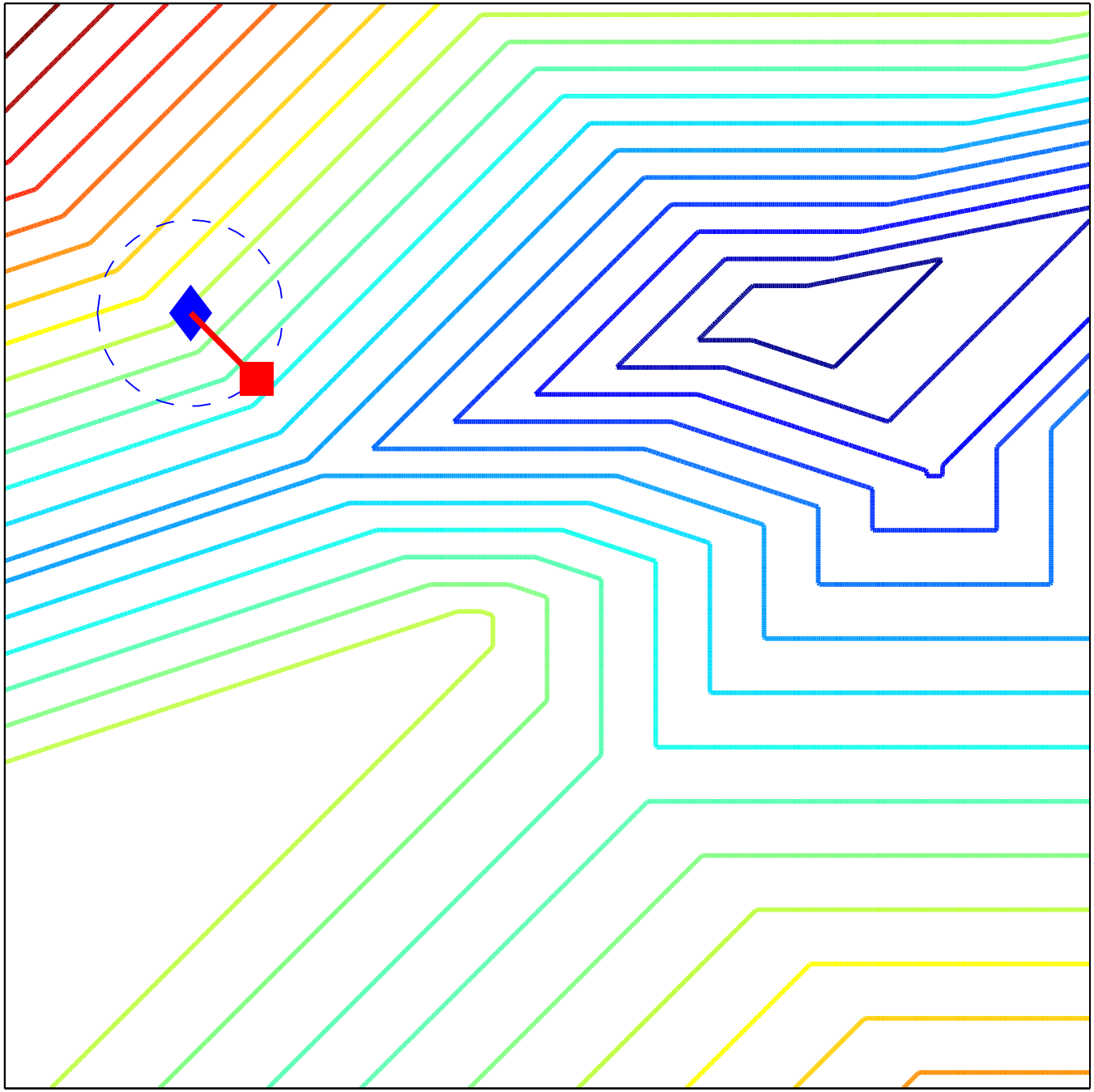


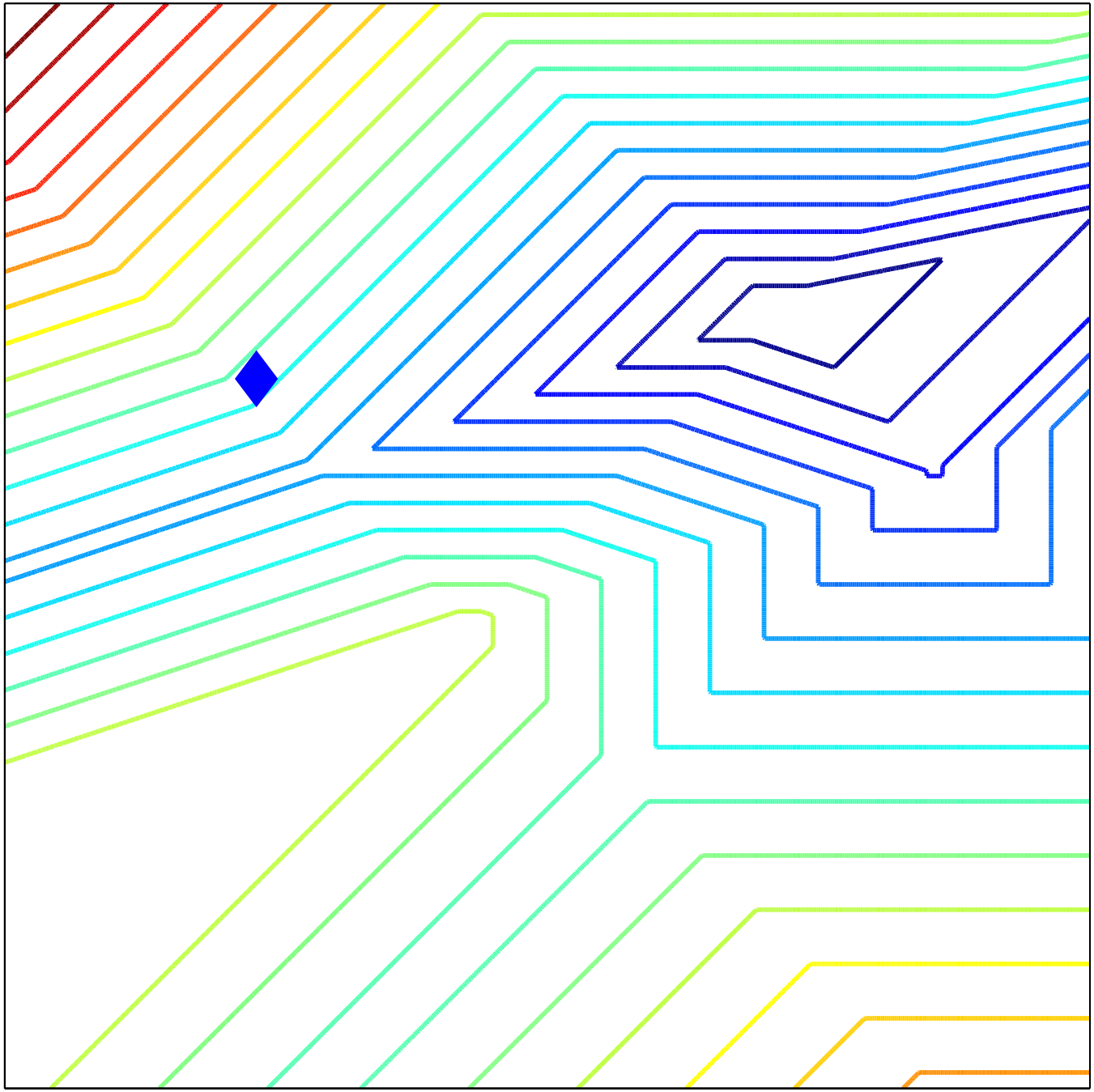


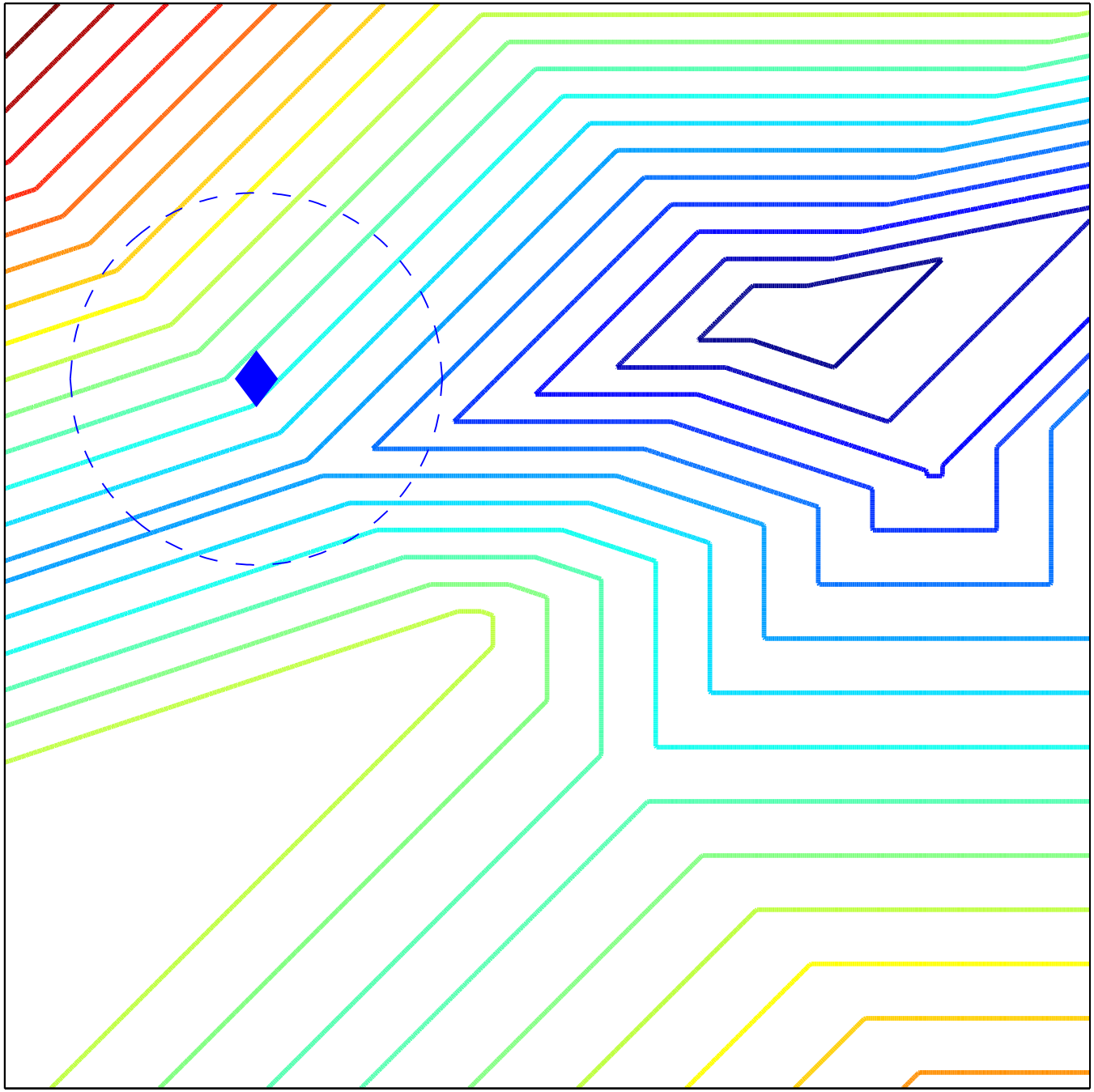


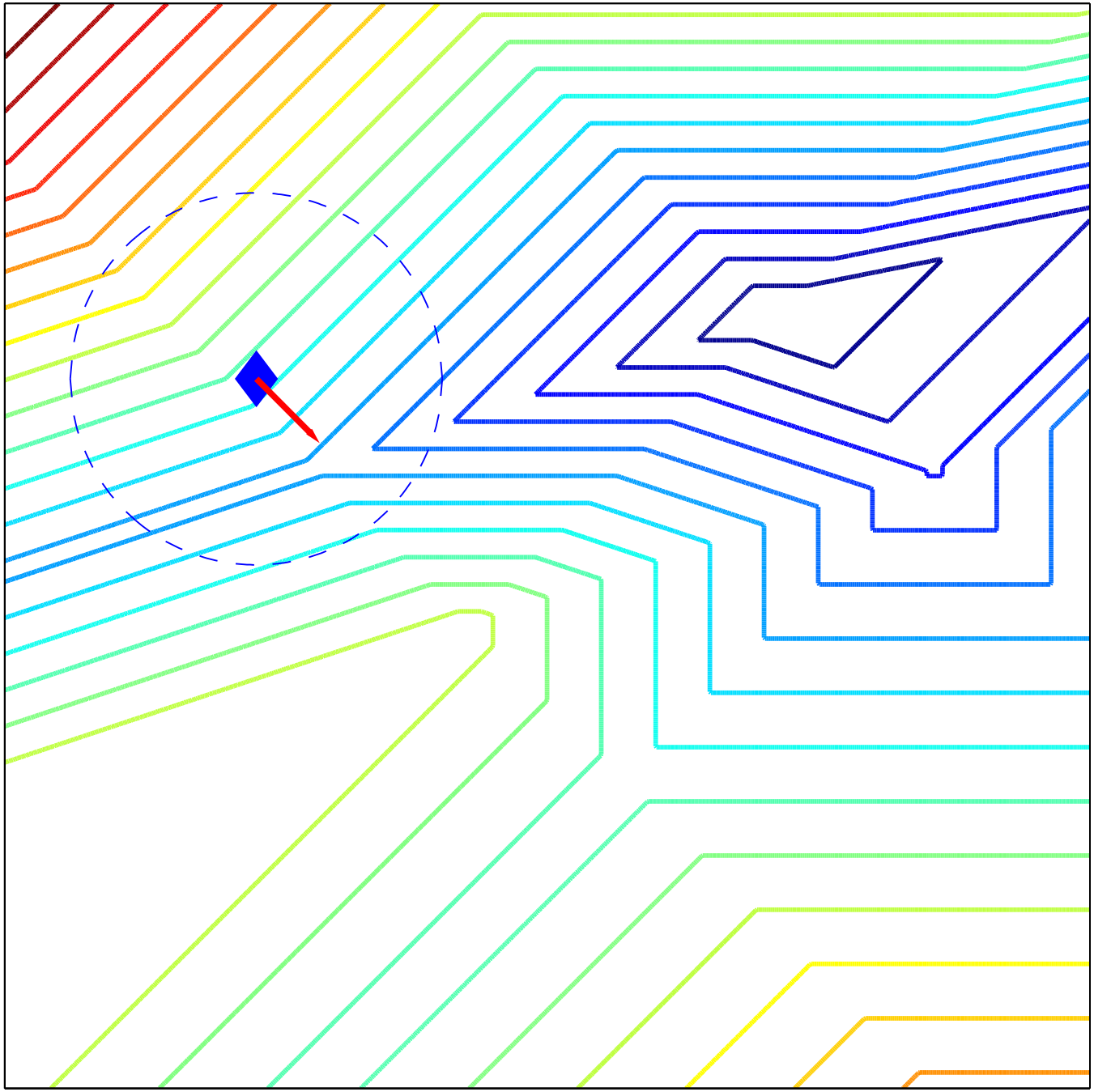


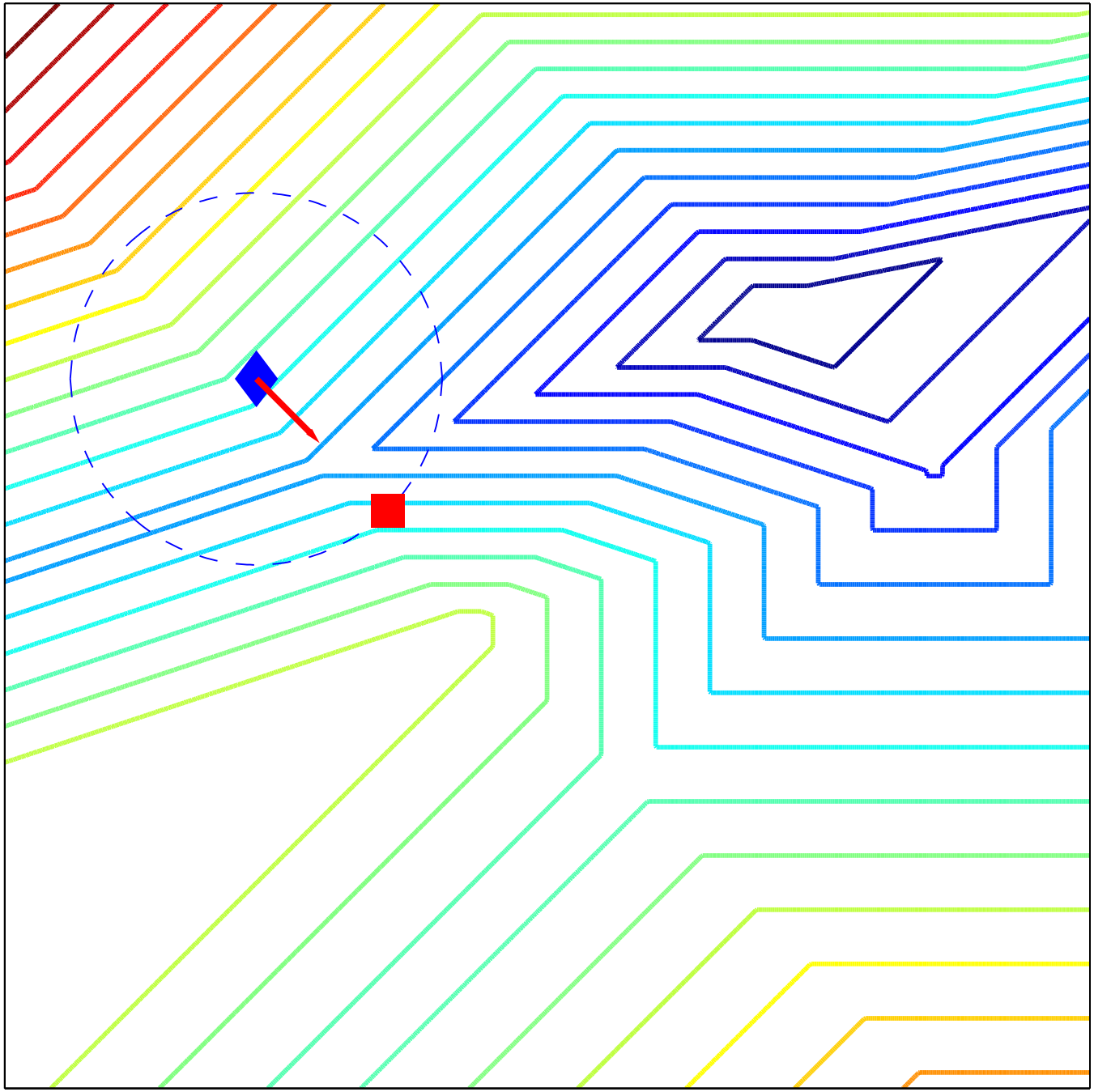


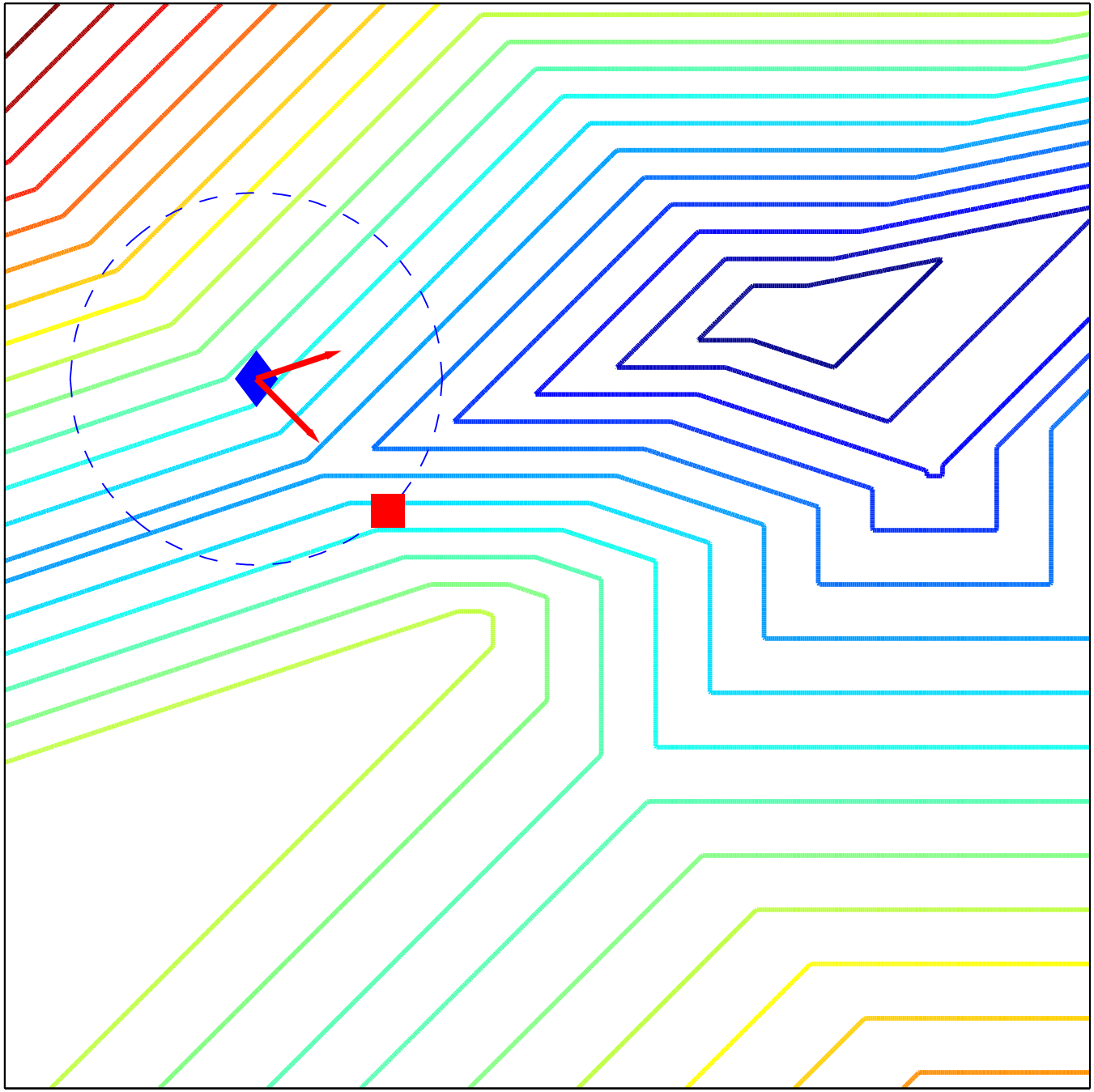


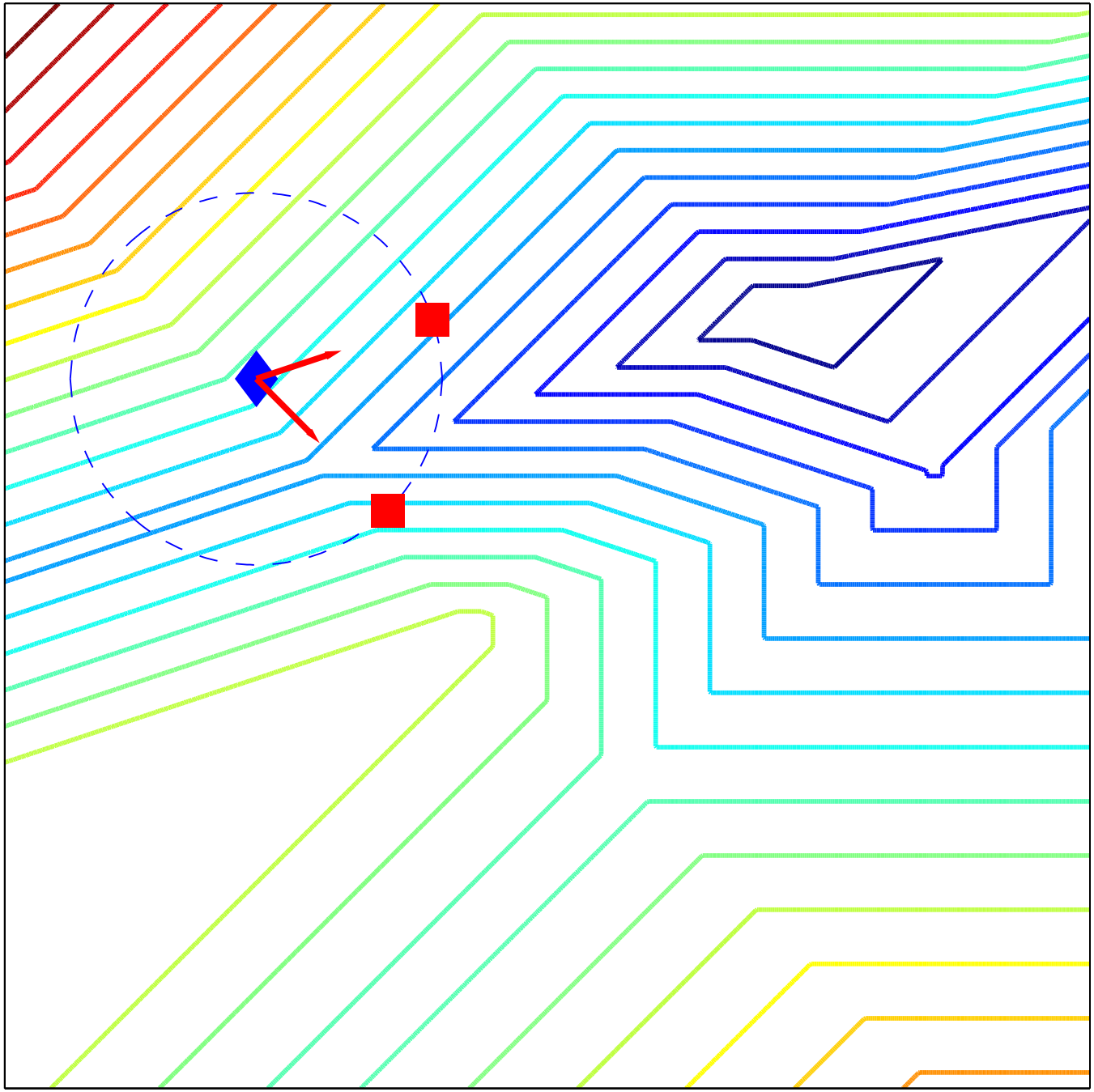












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At some iterate x^k,

$$\mathfrak{G}^k \triangleq \bigcup_{i \in I_h(F(x^k))} \left\{ \nabla \psi(x^k) + \nabla M(x^k) a_i \right\}$$

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Or, given a set of points $Y = \left\{ x^k, y^2, \ldots, y^p \right\} \subset \mathcal{B}(x^k, \Delta_k)$,

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Assumption

The set $\mathfrak{G}^k$ *satisfies* MS4PL-1 $\subseteq \mathfrak{G}^k \subseteq$ MS4PL-2.

Smooth master model

Our model gradients around iterate x^k satisfy

$$g^k \triangleq \mathbf{proj}\left(0, \mathbf{co}\left(\mathfrak{G}^k\right)\right) \in \mathbf{co}\left(\mathfrak{G}^k\right),$$

Let λ^* be the corresponding coefficients so that $g^k = G^k \lambda^*$.

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Define

$$A^k \triangleq \begin{bmatrix} | & & | \\ a_{j_1} & \cdots & a_{j_t} \\ | & & | \end{bmatrix},$$

and set $w^k = A^k\lambda^*$. Define the smooth ***master model*** $m_k^f : \mathbb{R}^n \to \mathbb{R}$,

$$m_k^f(x) \triangleq \psi(x) + \sum_{i=1}^{p} w_i^k m^{F_i}(x) + \sum_{i=1}^{p} \lambda_i^* b_{j_i}.$$

Trust region subproblem

Approximately solve

$$\begin{aligned} &\underset{s}{\text{minimize}}\ m_k^f(x^k + s) \\ &\text{subject to: } s \in \mathcal{B}(0, \Delta_k) \end{aligned}$$

to obtain a solution s satisfying

$$\psi(x^k) - \psi(x^k + s) + \left\langle M(x^k) - M(x^k + s), w^k \right\rangle \geq \frac{\kappa_{\mathrm{d}}}{2} \|g^k\| \min\left\{ \Delta_k, \frac{\|g^k\|}{\kappa_{\mathrm{mh}}} \right\}.$$

Measuring descent

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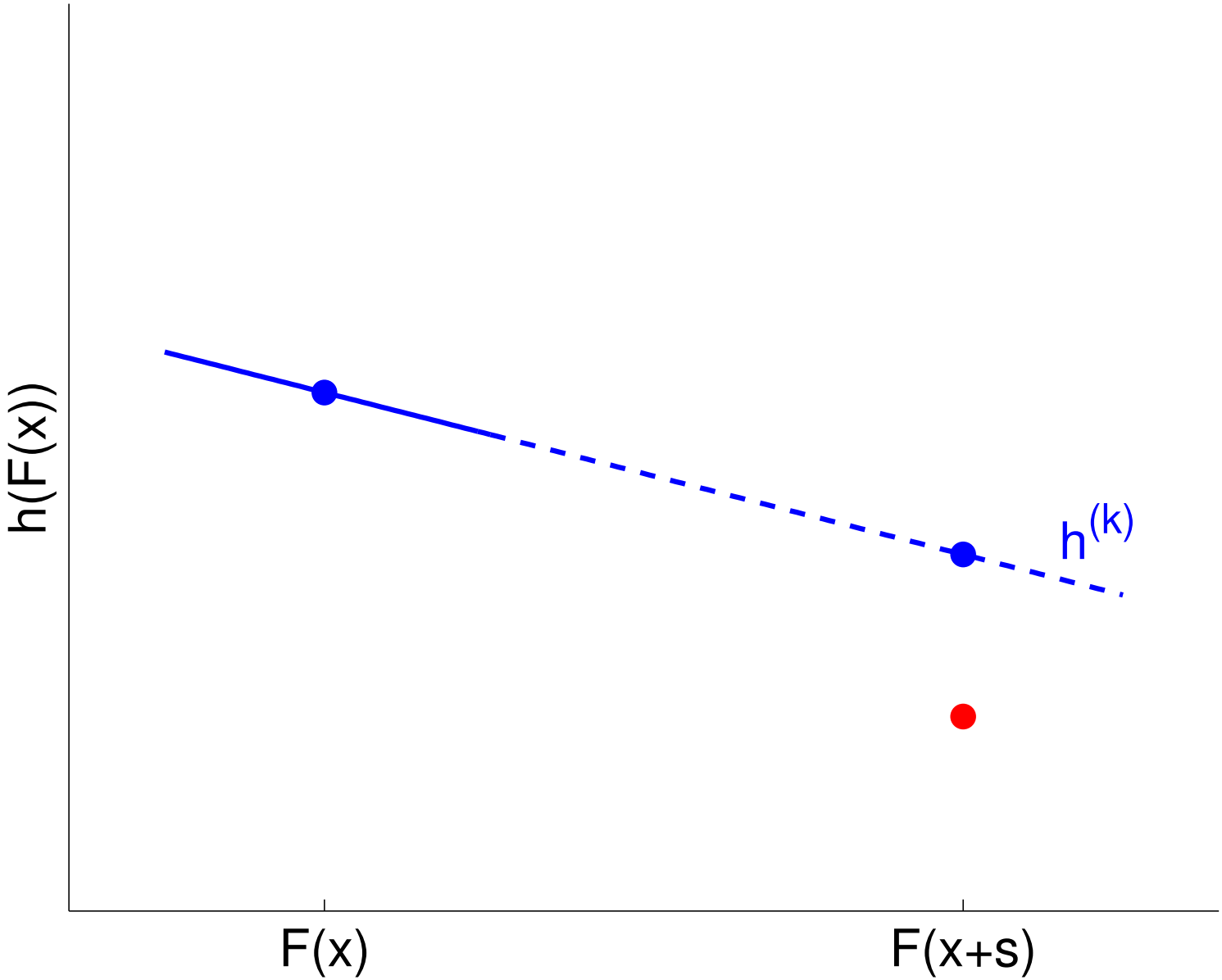
- $$\rho_k \triangleq \frac{\psi(x^k) - \psi(x^k + s^k) + h^{(k)}(F(x^k)) - h^{(k)}(F(x^k + s^k))}{\psi(x^k) - \psi(x^k + s^k) + \langle M(x^k) - M(x^k + s^k), a^{(k)} \rangle}$$

Examples of $h^{(k)}$



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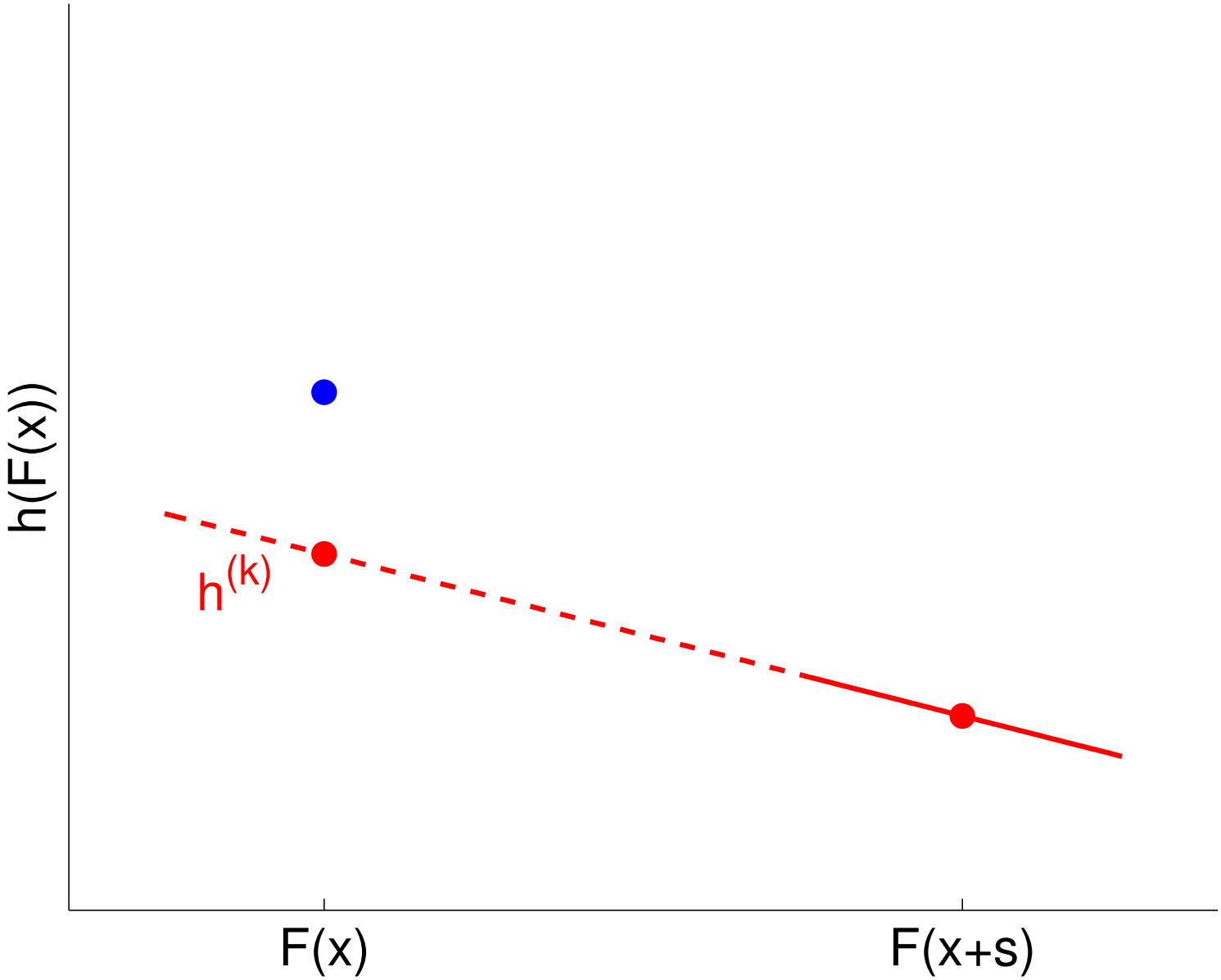

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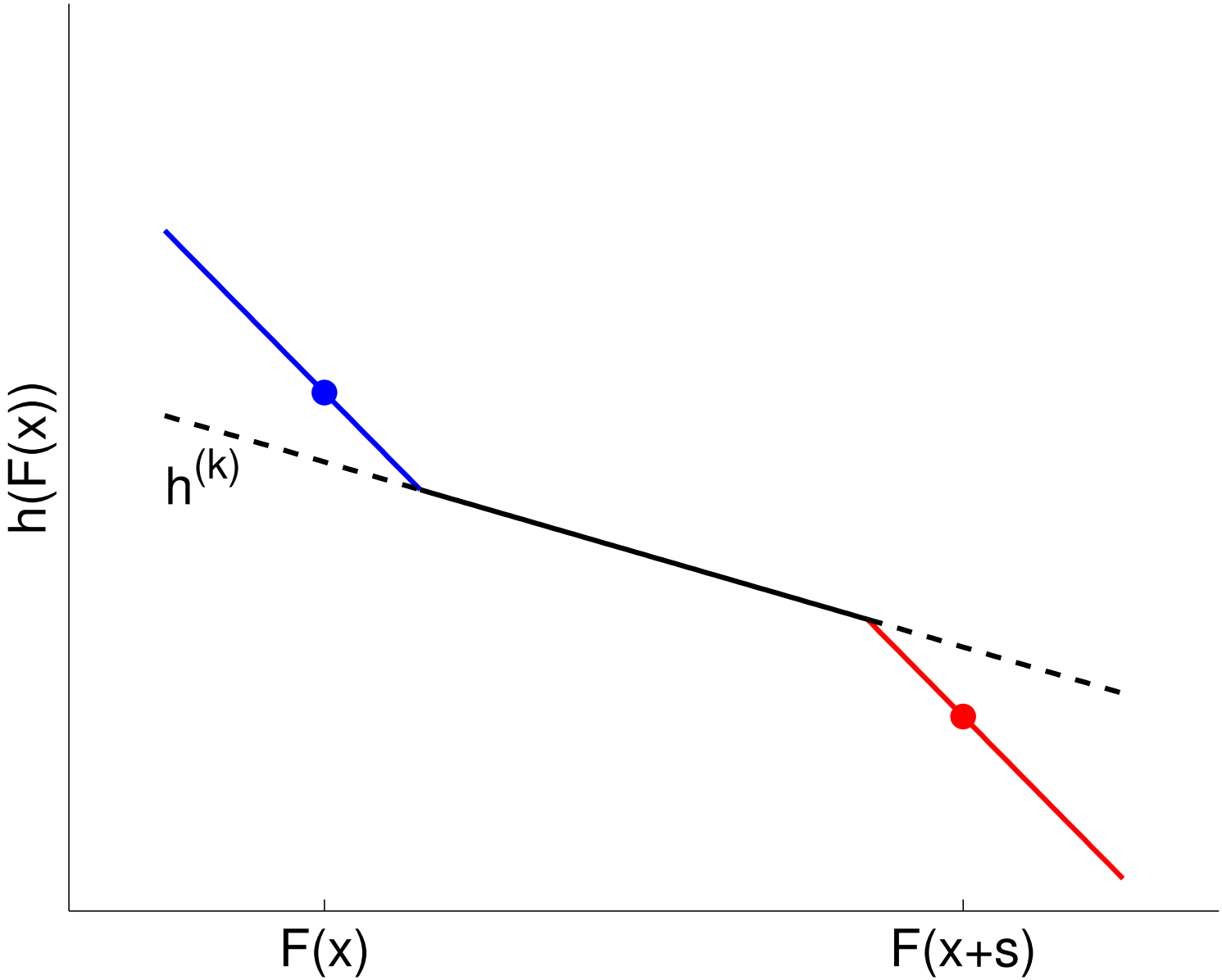

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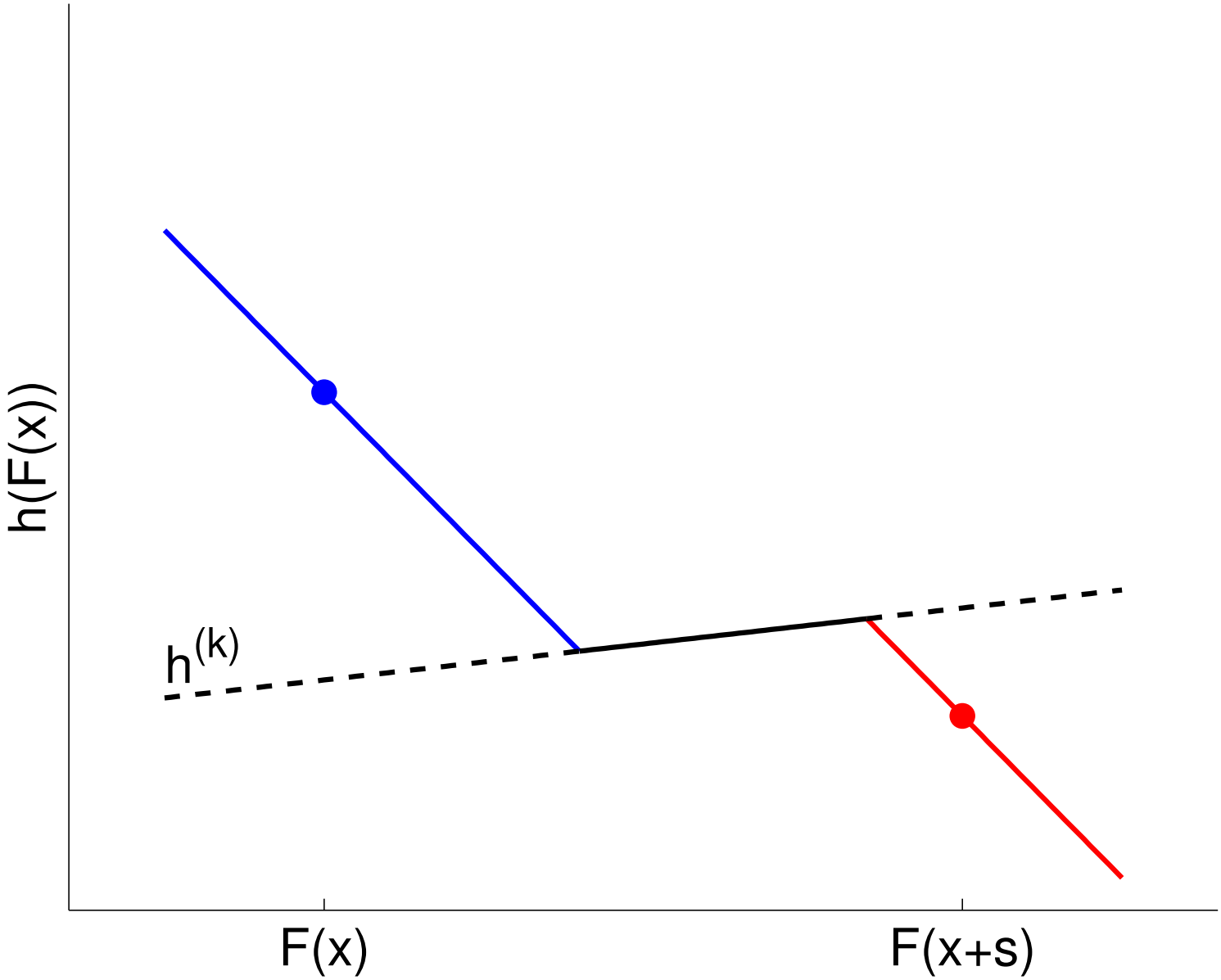

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- Measuring descent with ρ_k

Algorithm MS4PL

Choose x^0 **and** Δ_0
for $k = 0, 1, 2, \ldots$ **do**
  Build p **component models** m^{F_i} **fully linear on** $\mathcal{B}(x^k, \Delta_k)$
  Form $\nabla M(x^k)$ **using** $\nabla m^{F_i}(x^k)$ **and construct** $\mathfrak{G}^k \subset \mathbb{R}^n$
  $\rho_k \leftarrow -\infty$
  while $\rho_k = -\infty$ **do**
    if $\Delta_k < \eta_2 \|\nabla m^f(x^k)\|$ **then**
      Approximately solve TRSP to obtain s^k
      Evaluate $F(x^k + s^k)$ **and find** $h^{(k)}$
      if $(\nabla \psi(x^k) + \nabla M(x^k)\, a^{(k)}) \in \mathfrak{G}^k$ **then**
        Calculate ρ_k
      else
        $\mathfrak{G}^k \leftarrow \mathfrak{G}^k \cup \{\nabla \psi(x^k) + \nabla M(x^k)\, a^{(k)}\}$
        Update component models m^{F_i} **and master model** m^f
    else
      break
  if $\rho_k > \eta_1 > 0$ **then**
    $x^{k+1} \leftarrow x^k + s^k$, $\Delta_{k+1} \leftarrow \min\{\gamma_{\text{inc}} \Delta_k, \Delta_{\max}\}$
  else
    $x^{k+1} \leftarrow x^k$, $\Delta_{k+1} \leftarrow \gamma_{\text{dec}} \Delta_k$

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At some iterate x^k,

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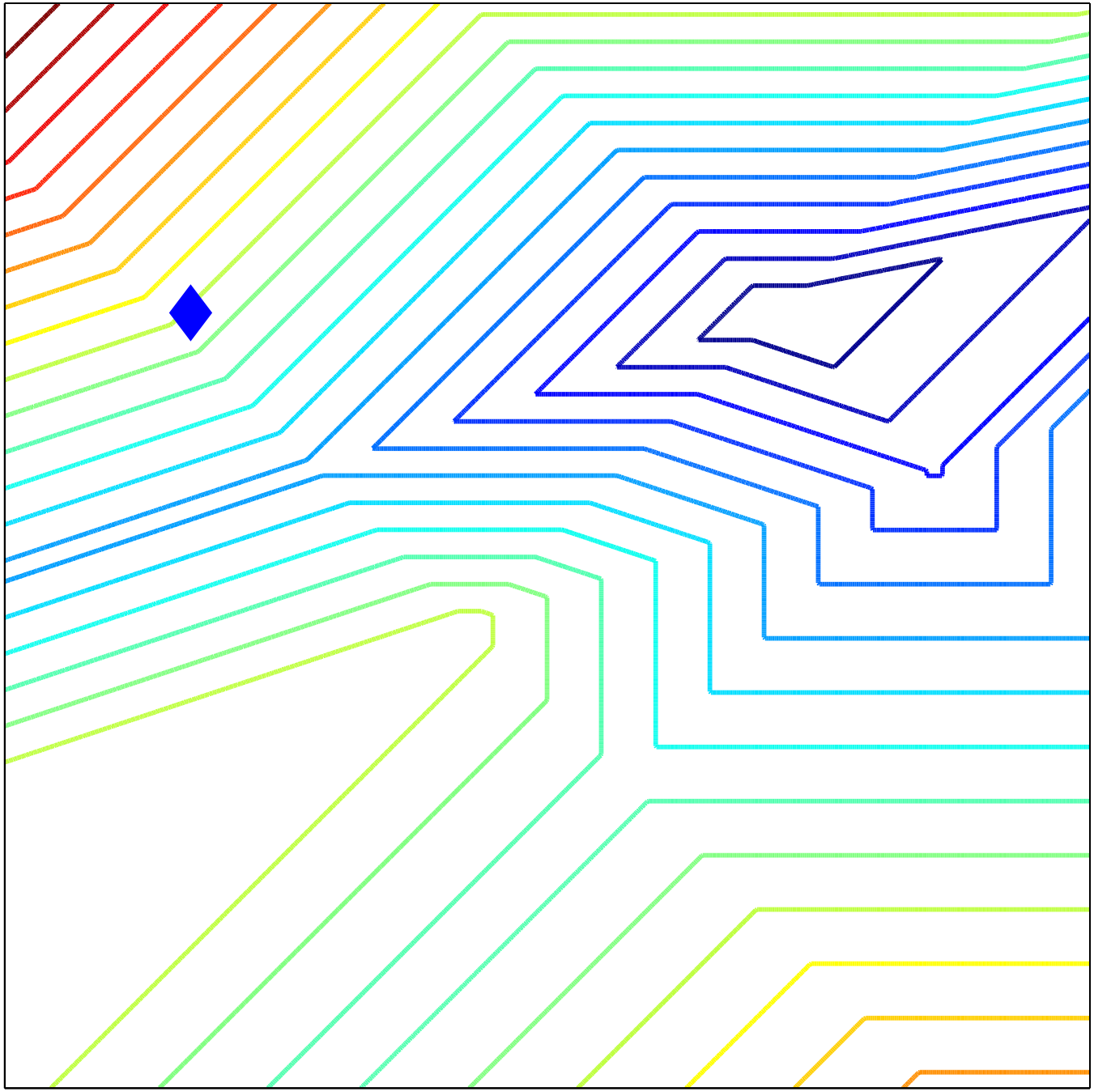
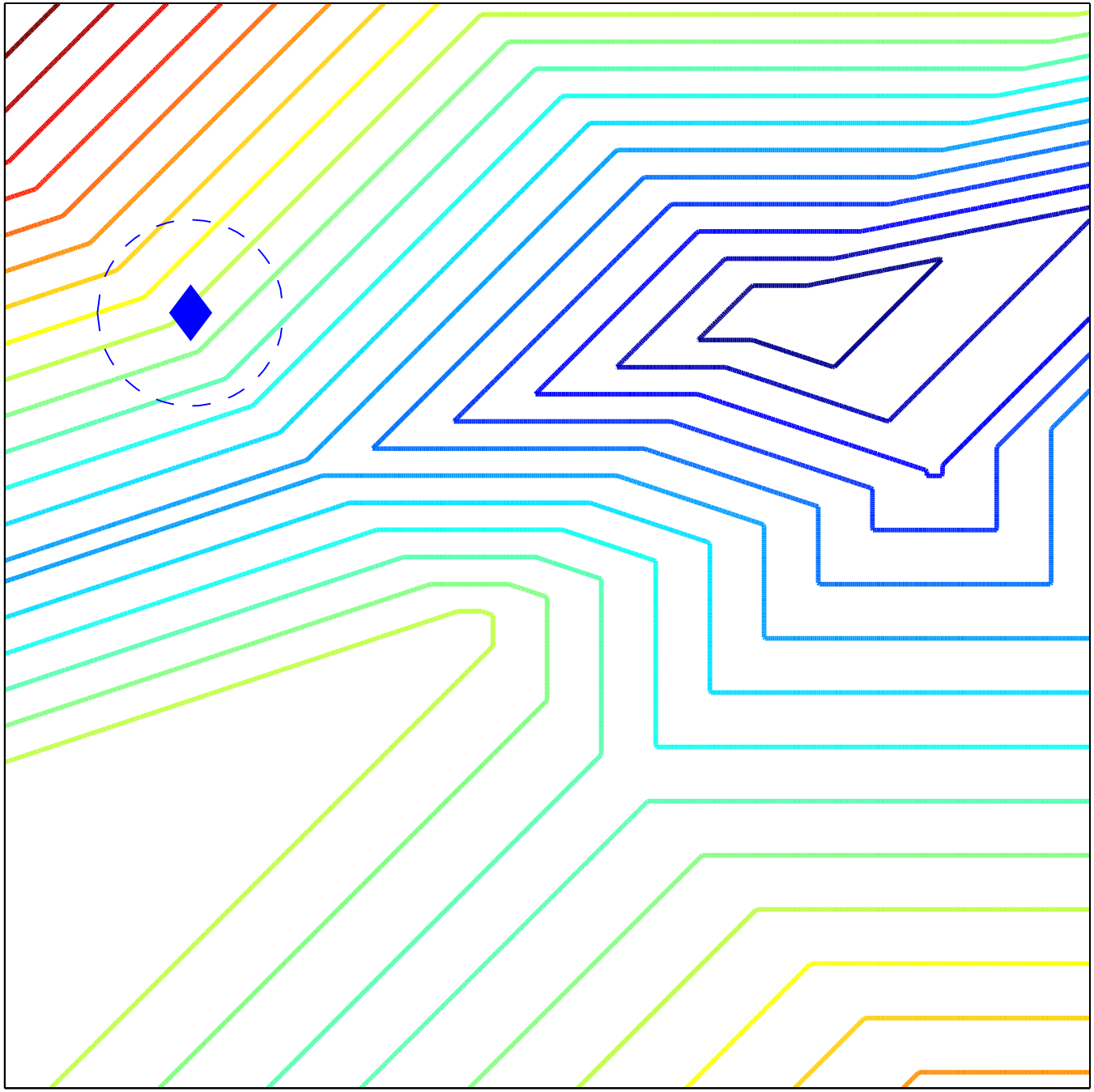
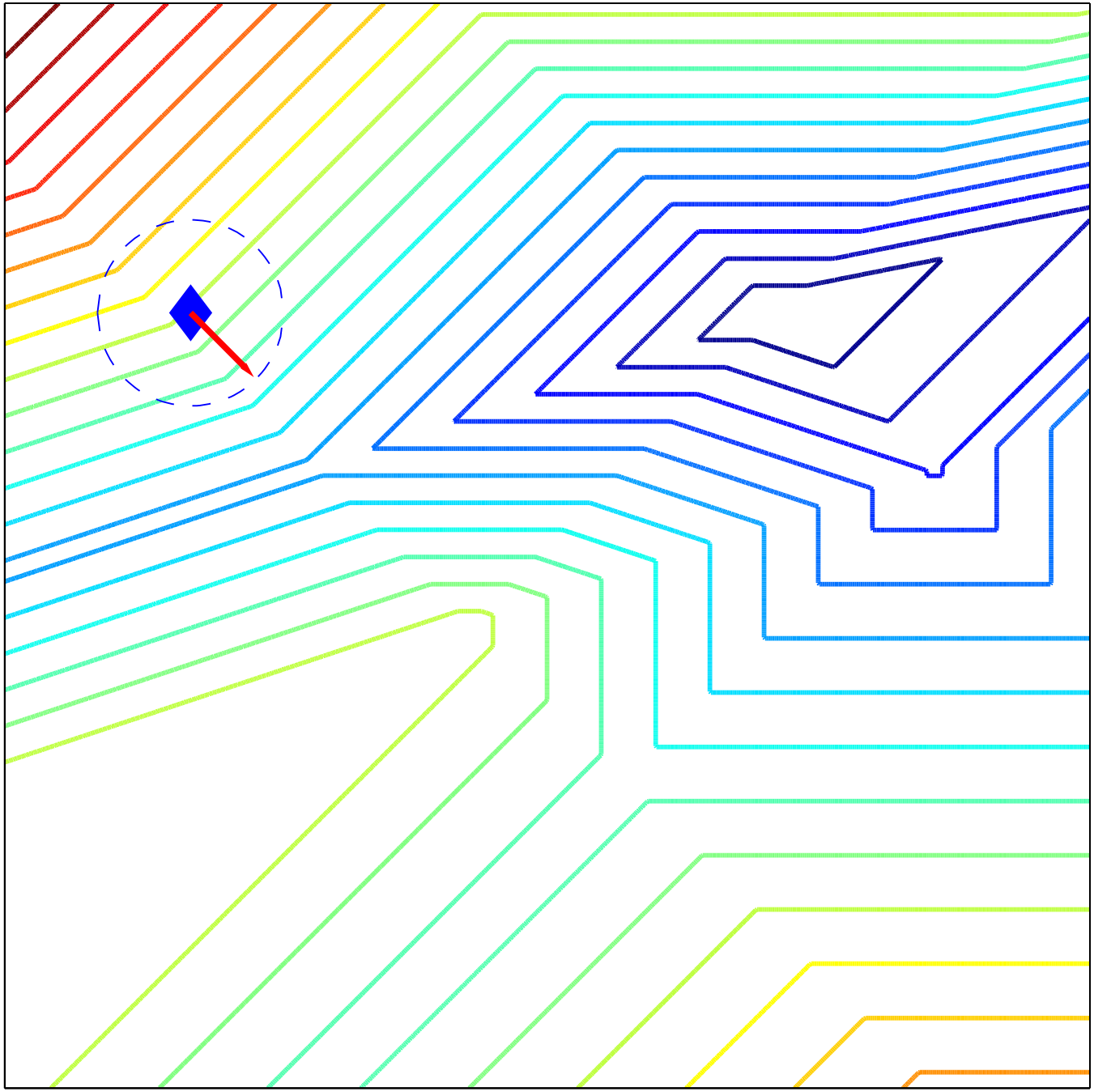
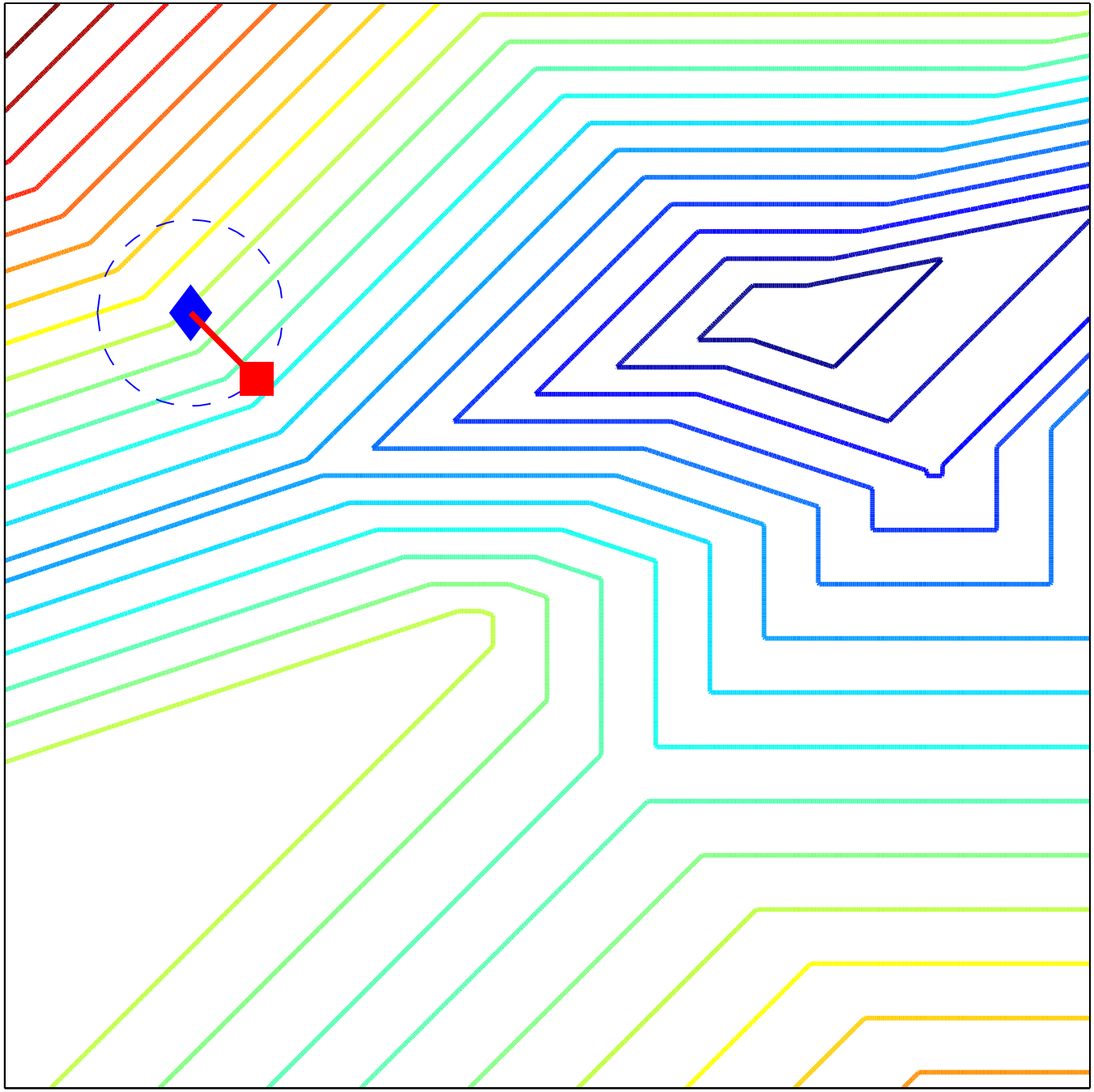
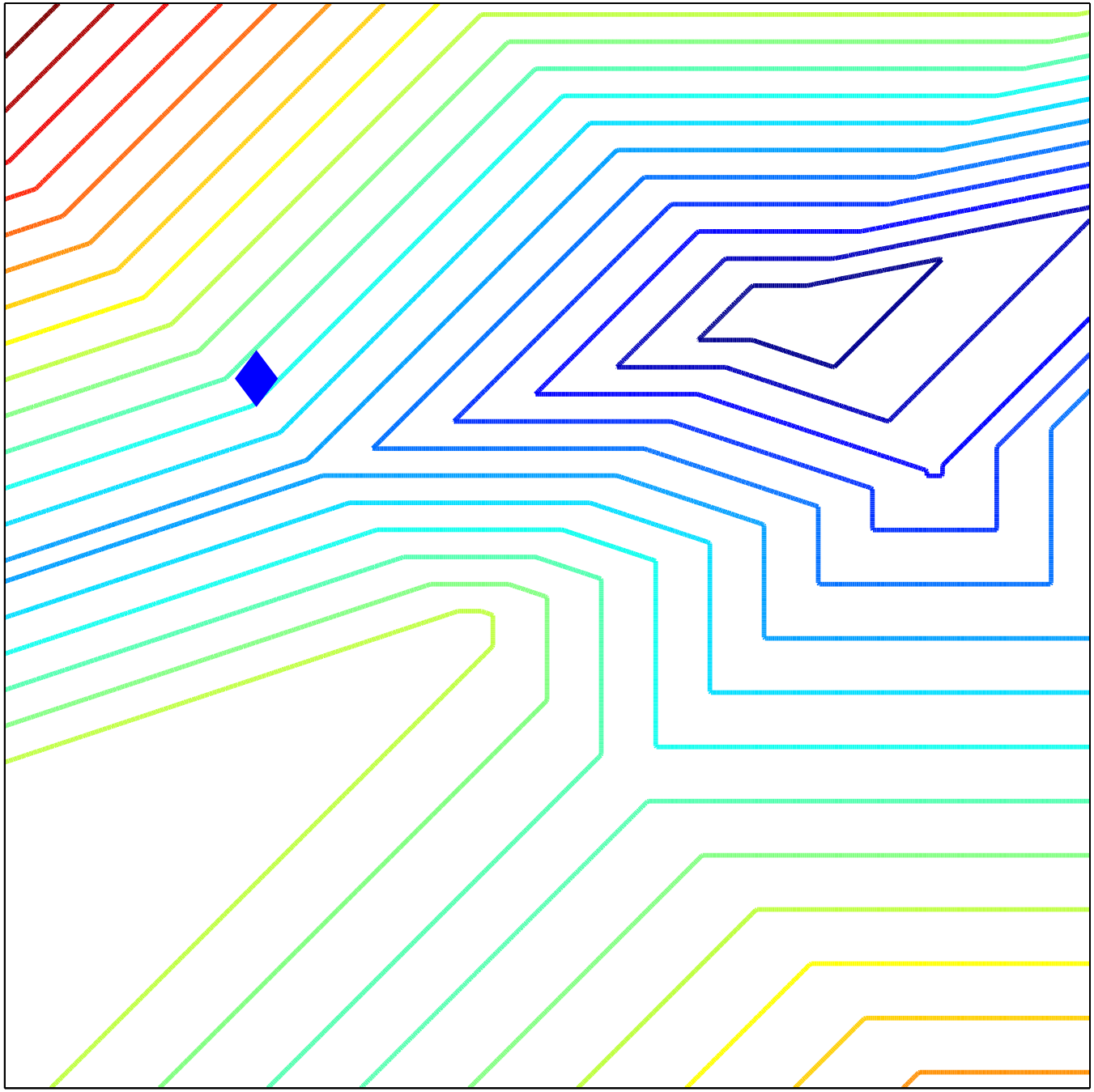
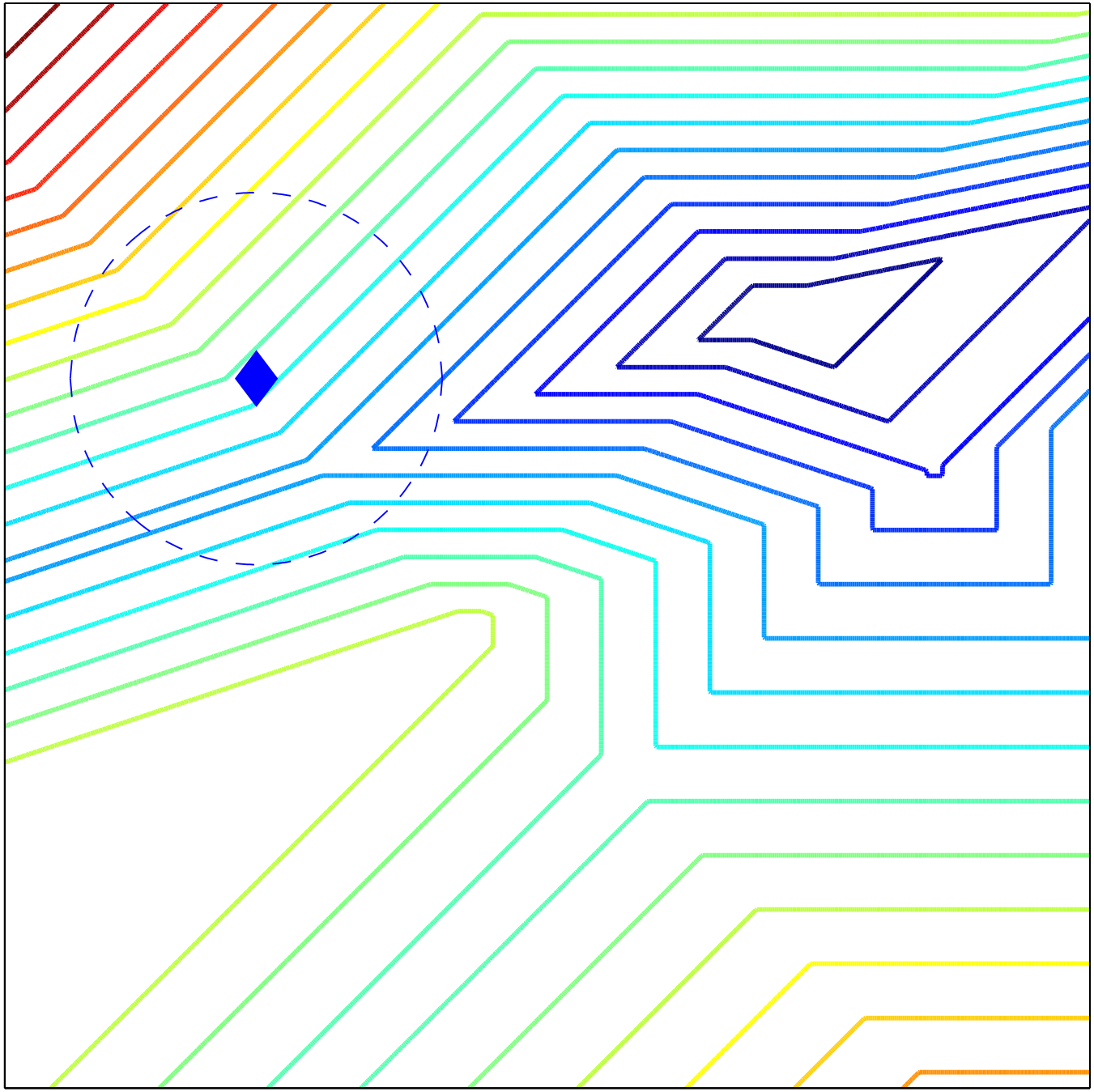


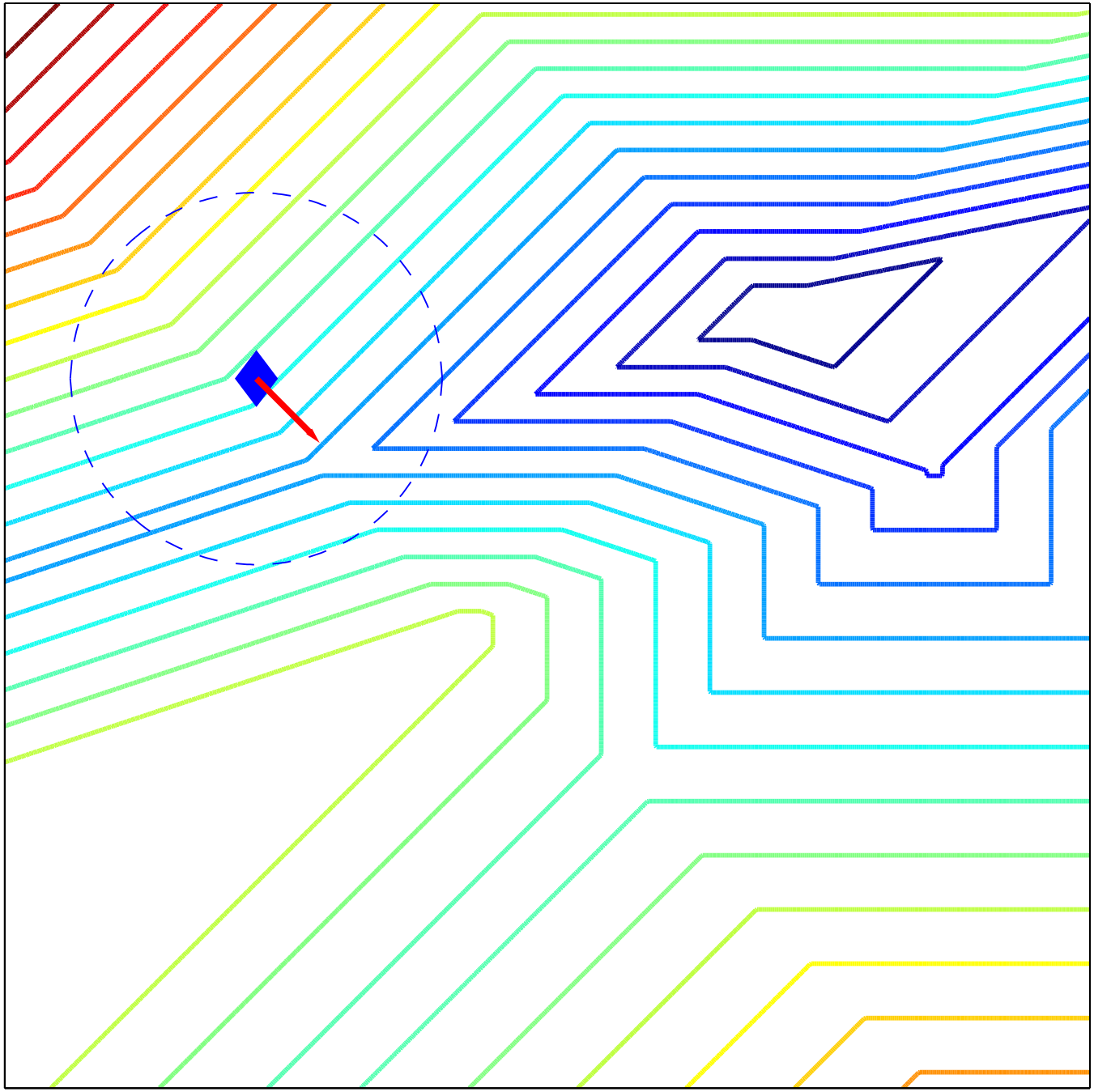


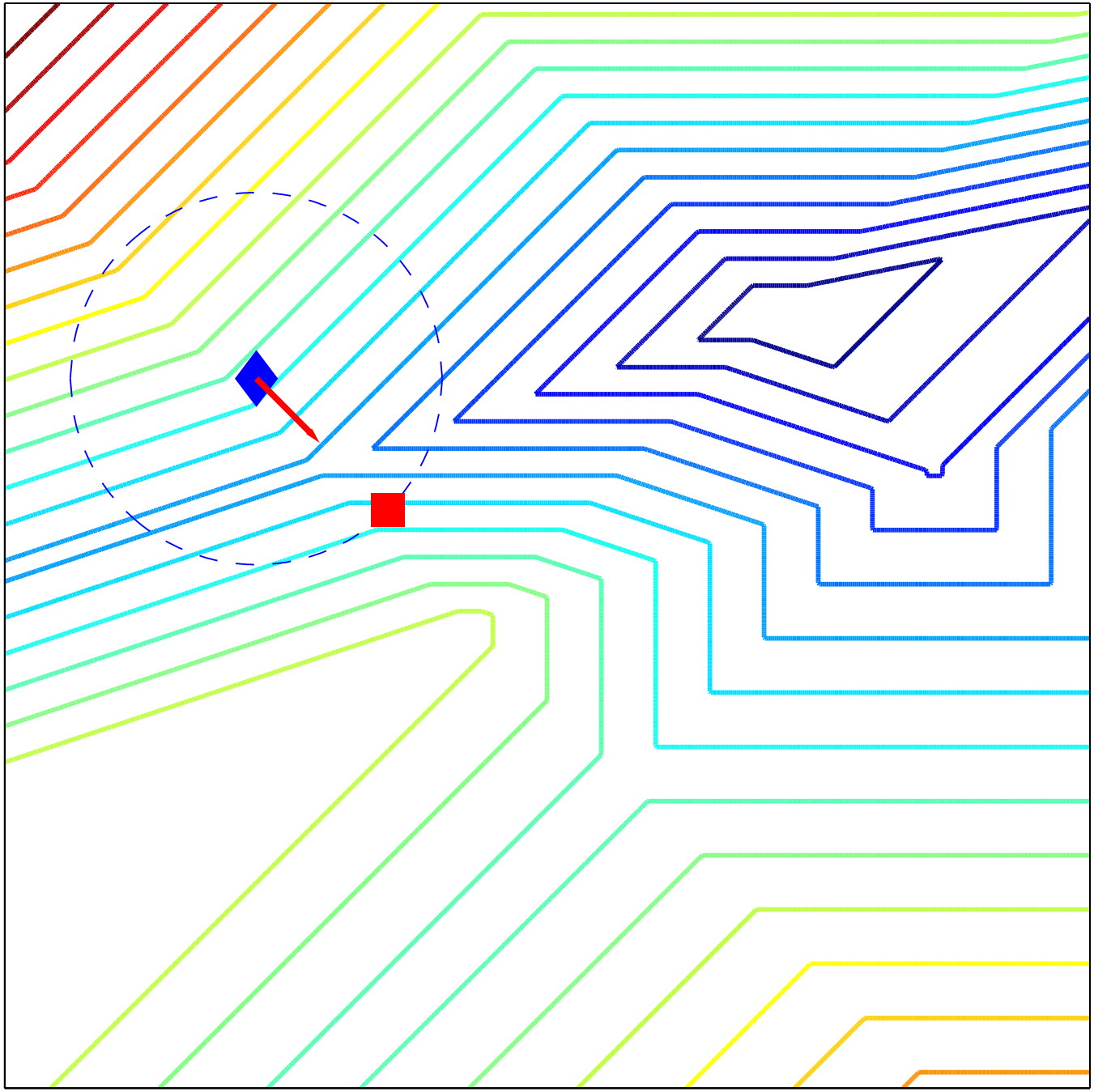


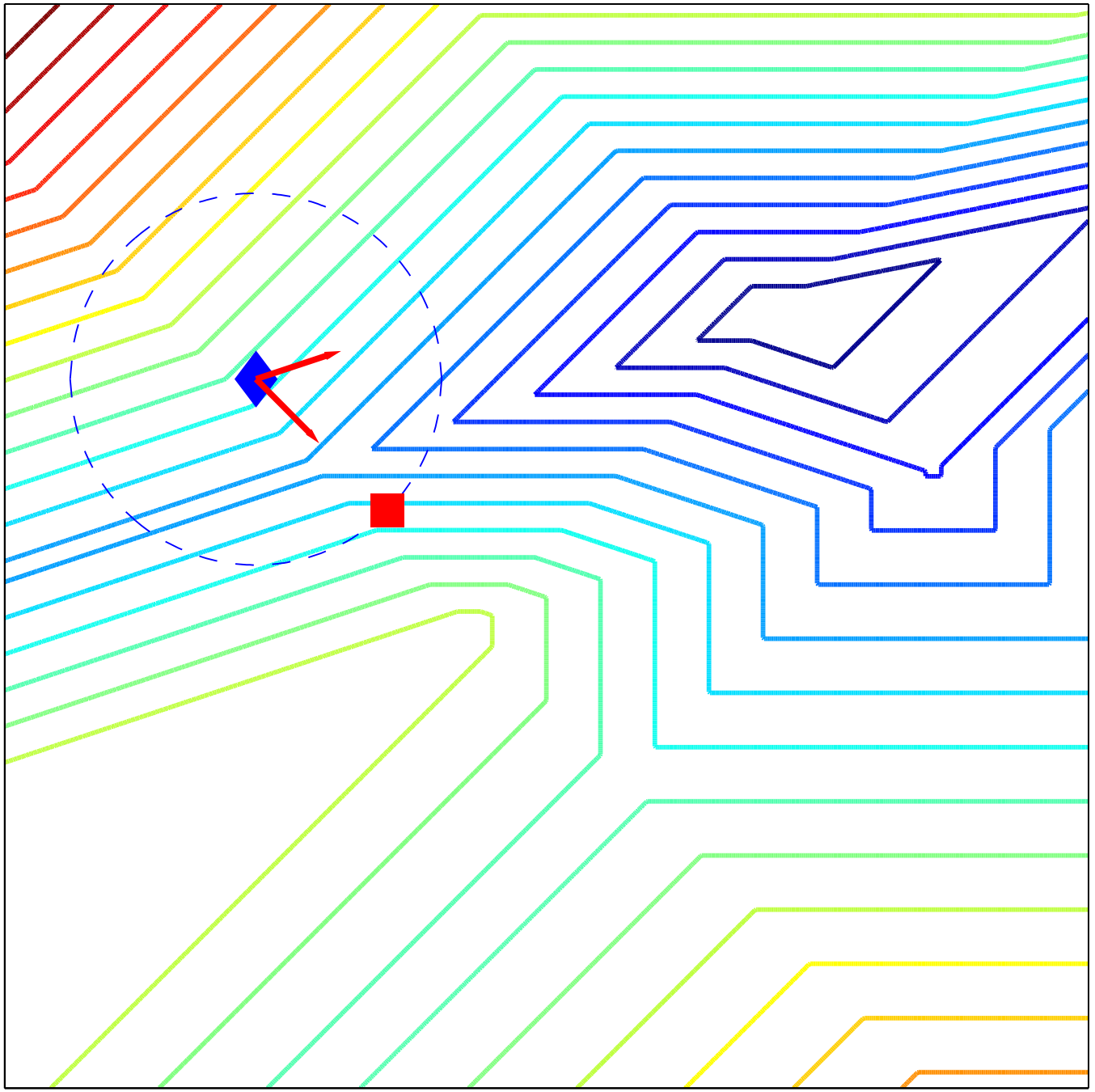


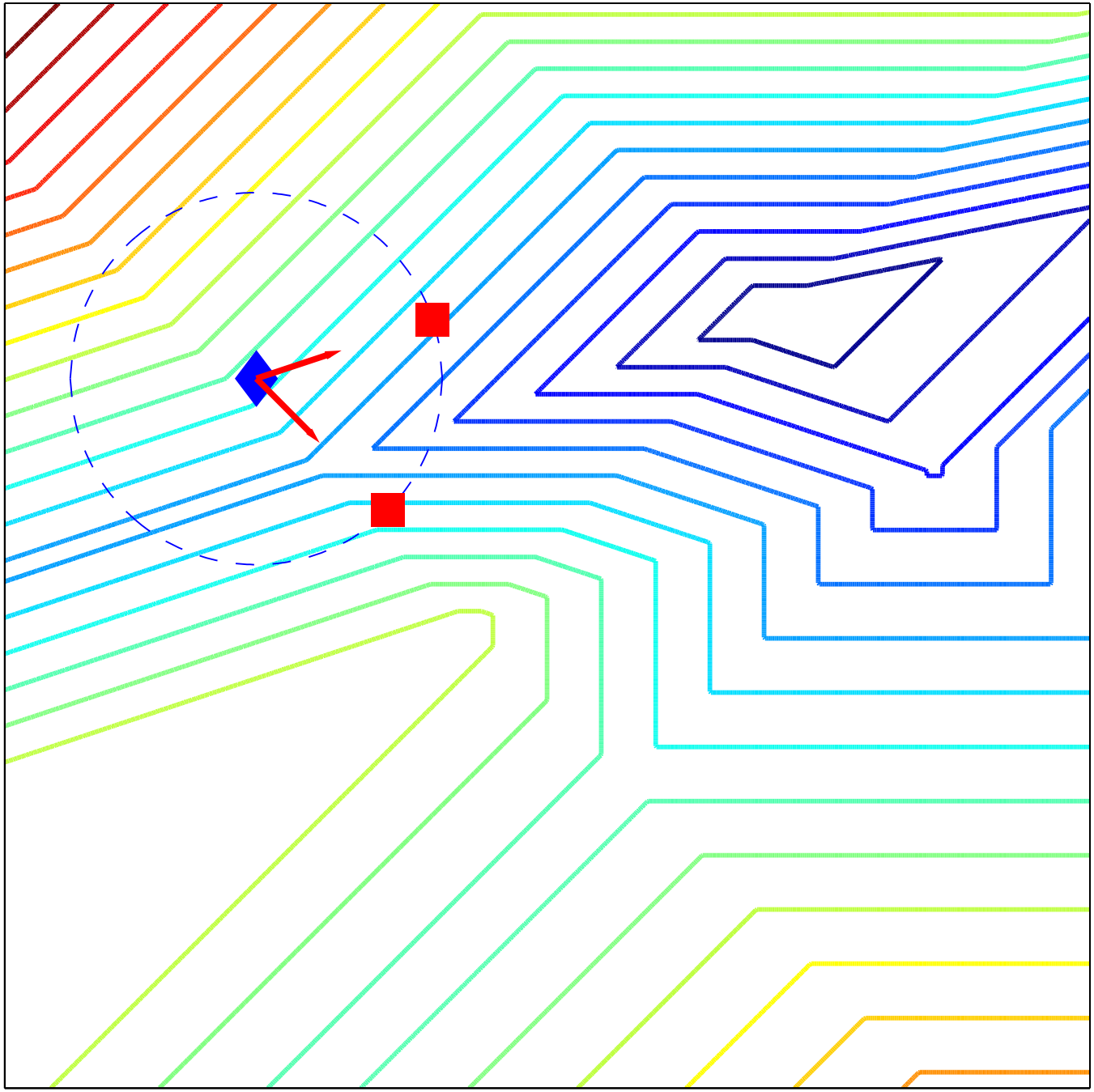


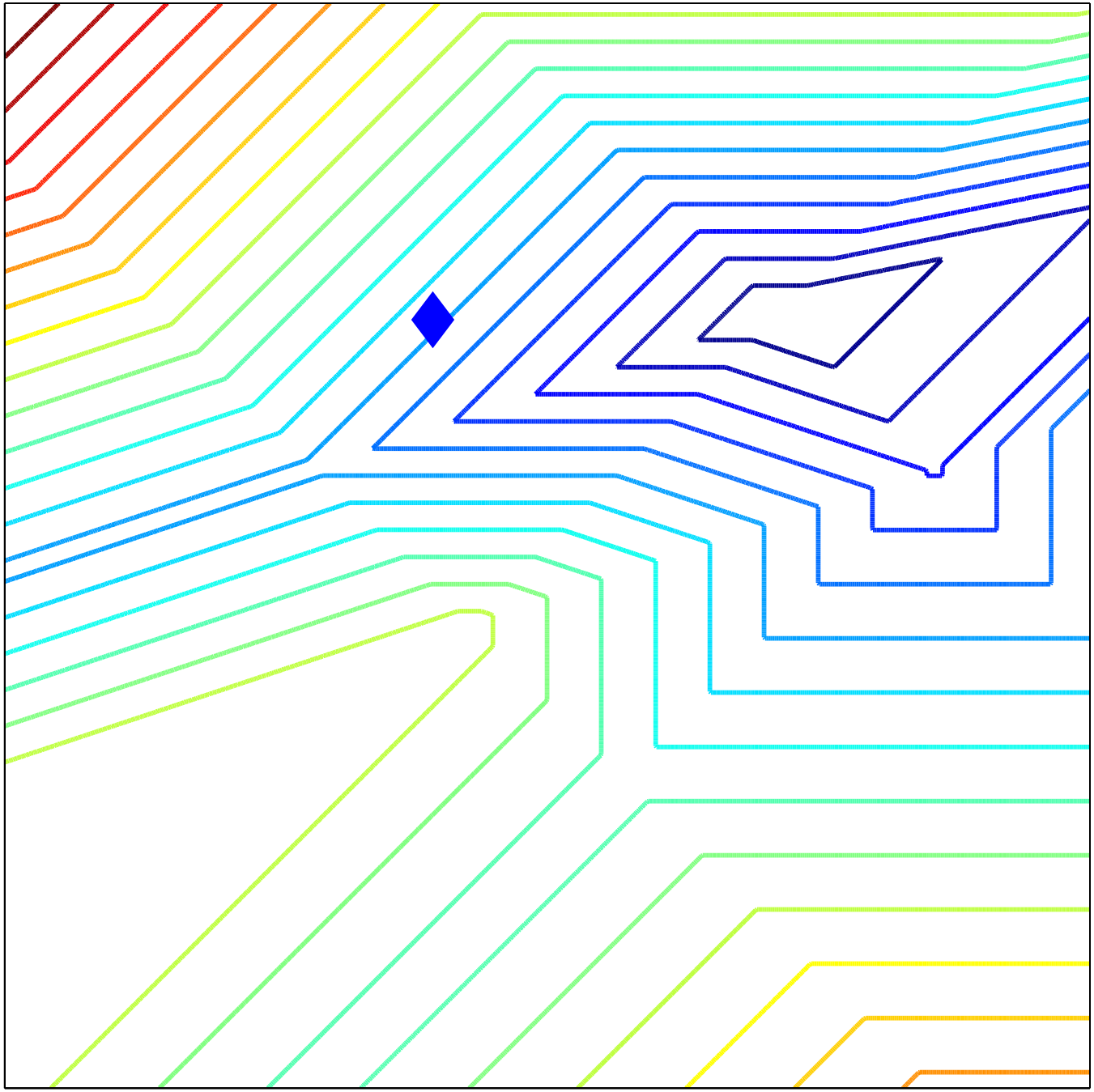


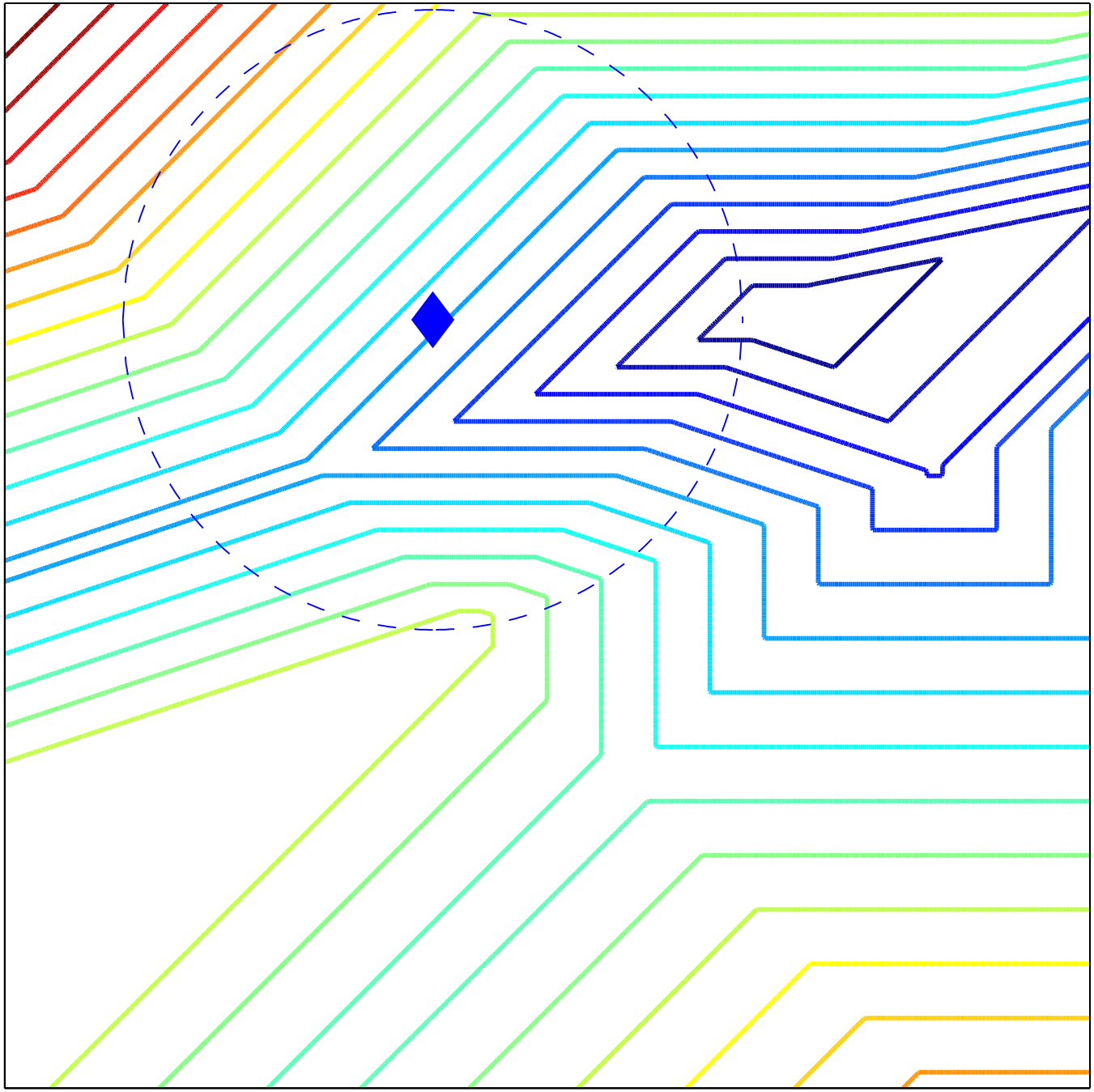


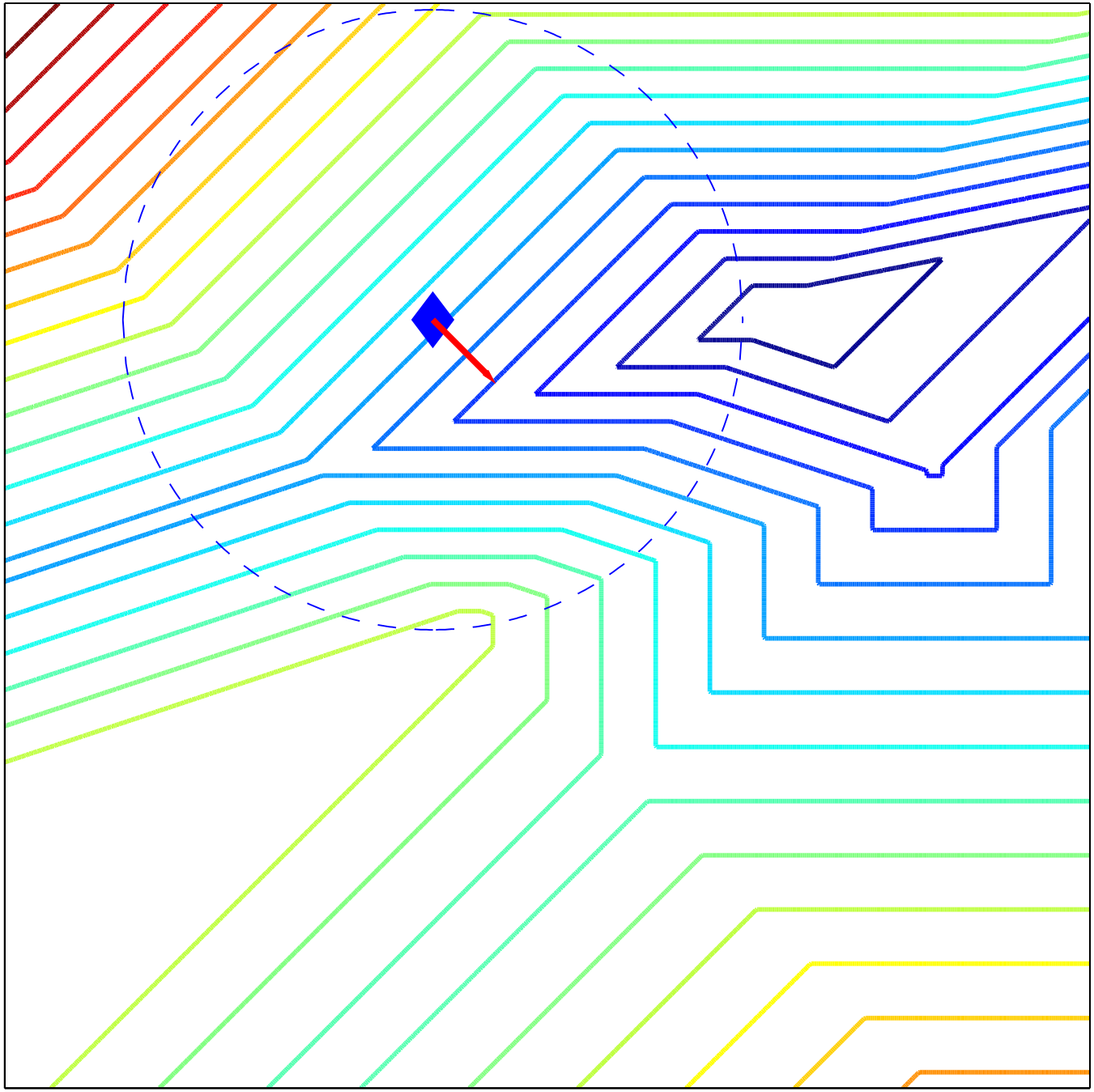


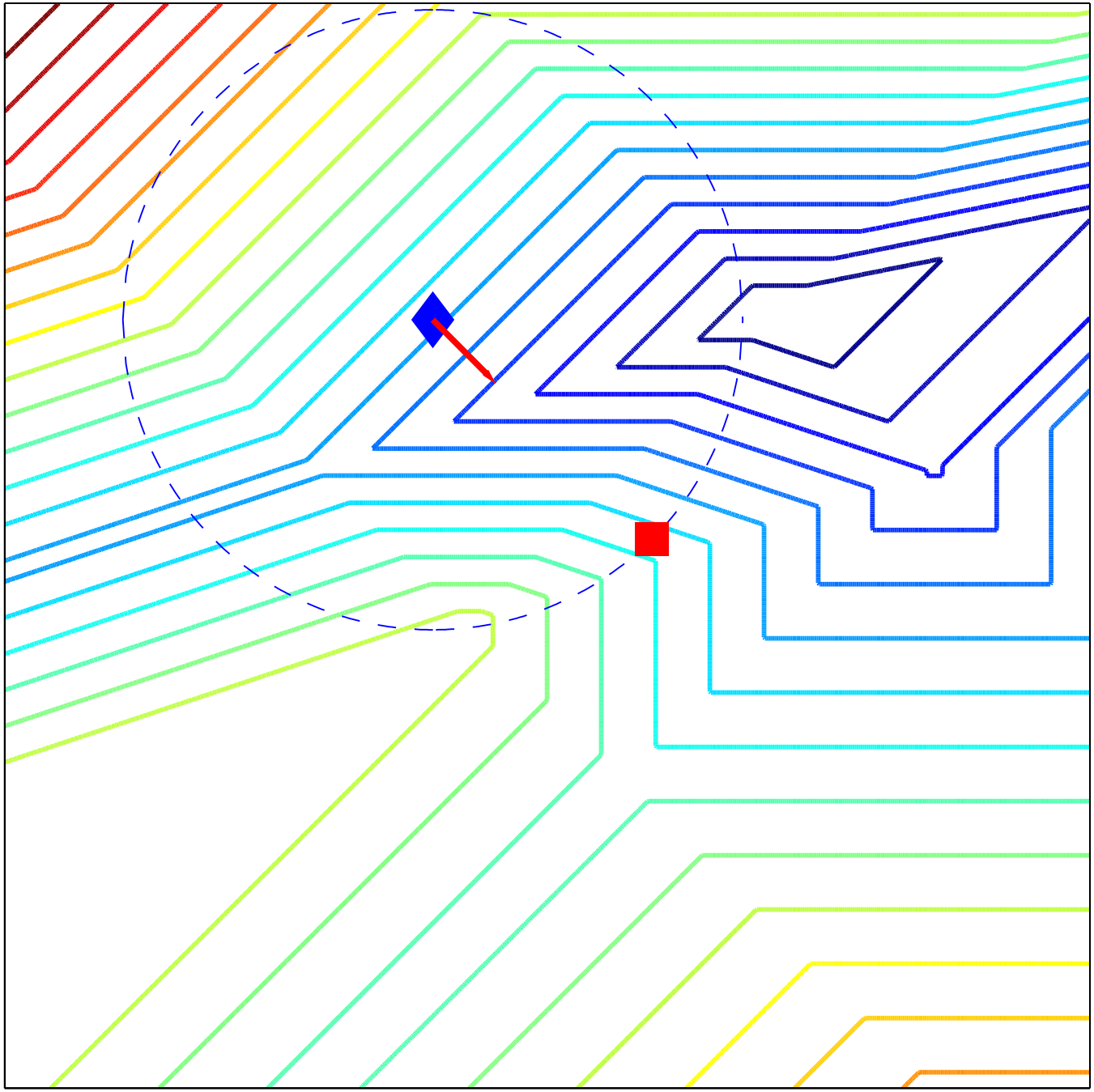


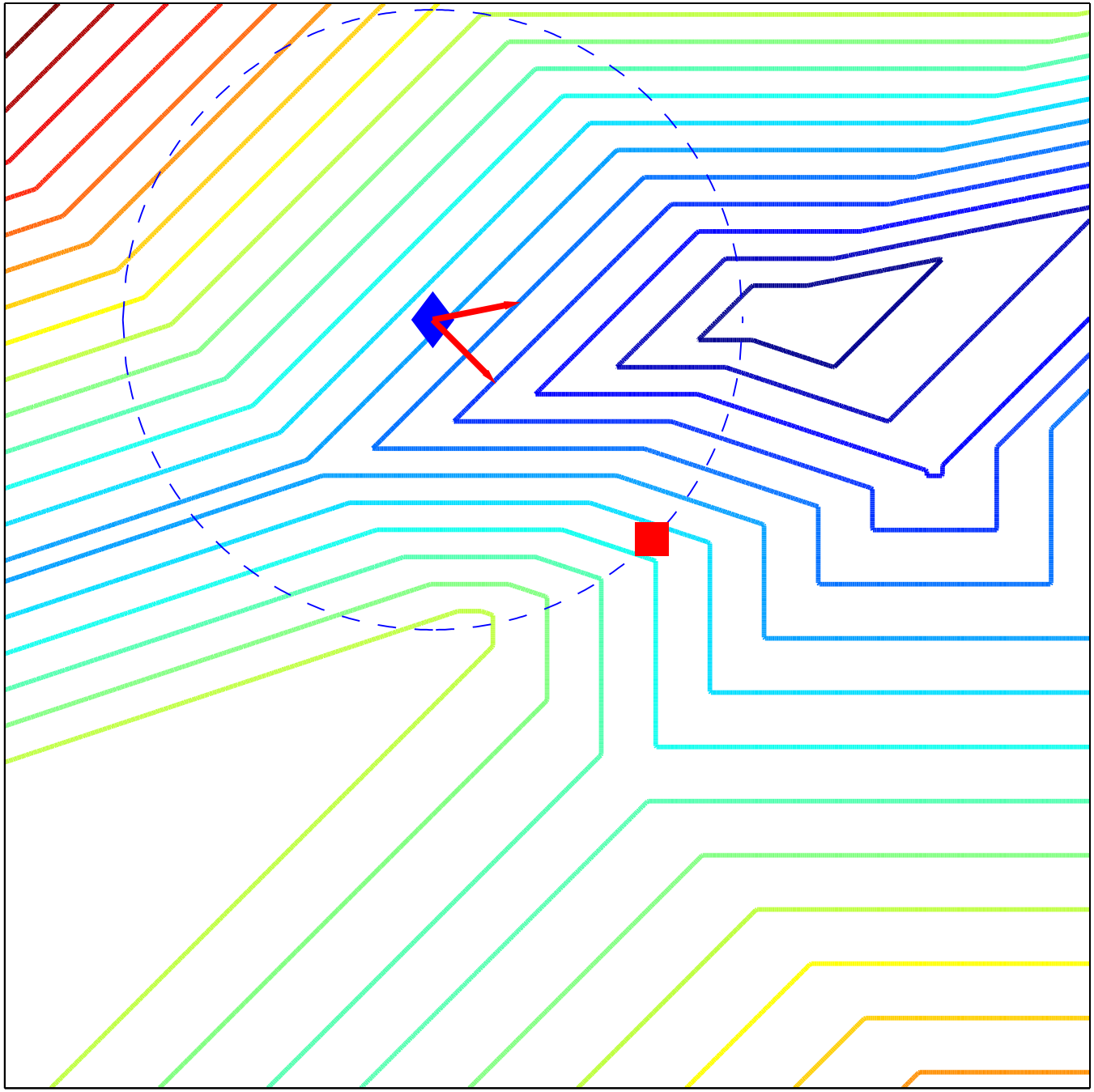


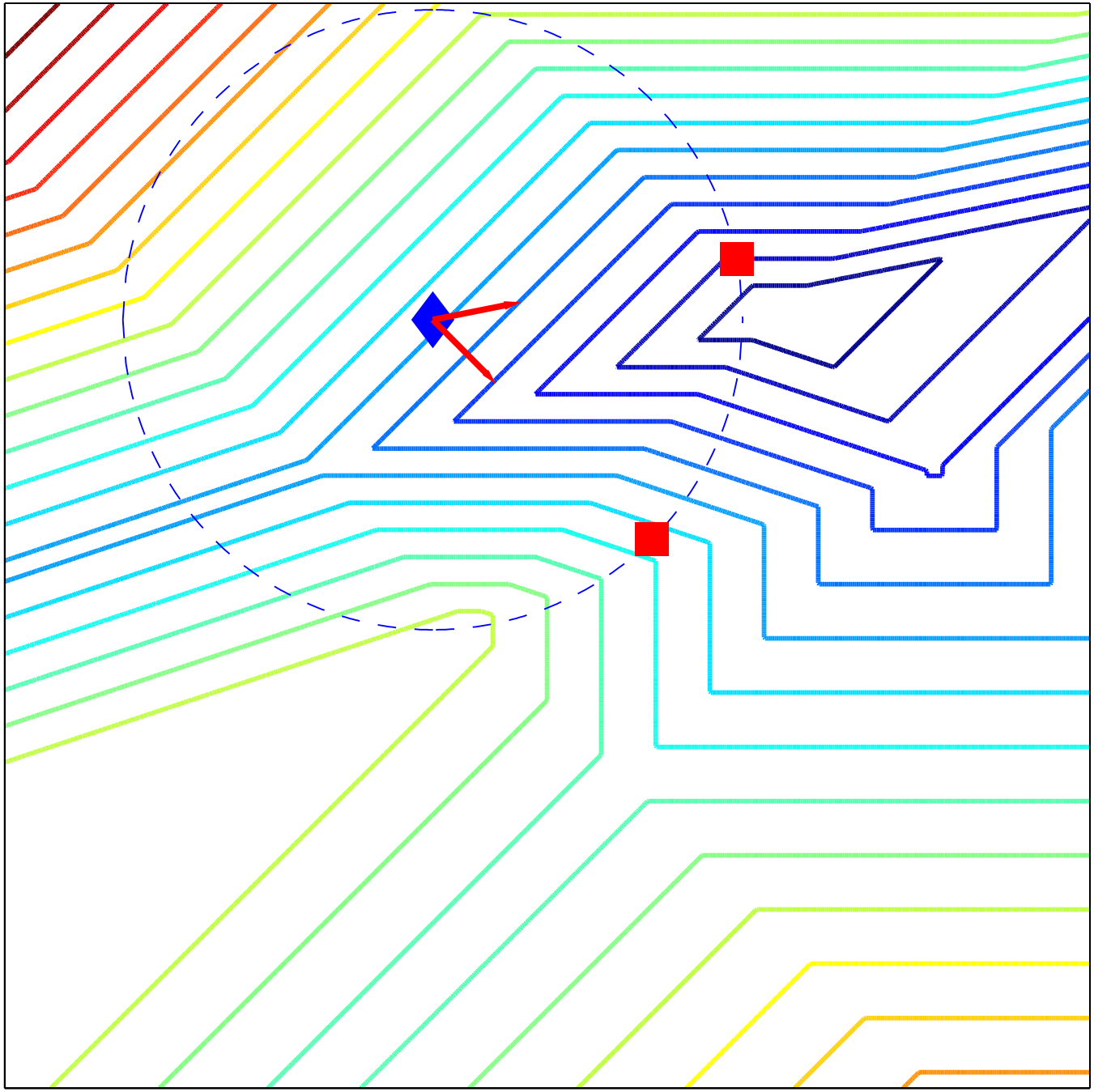


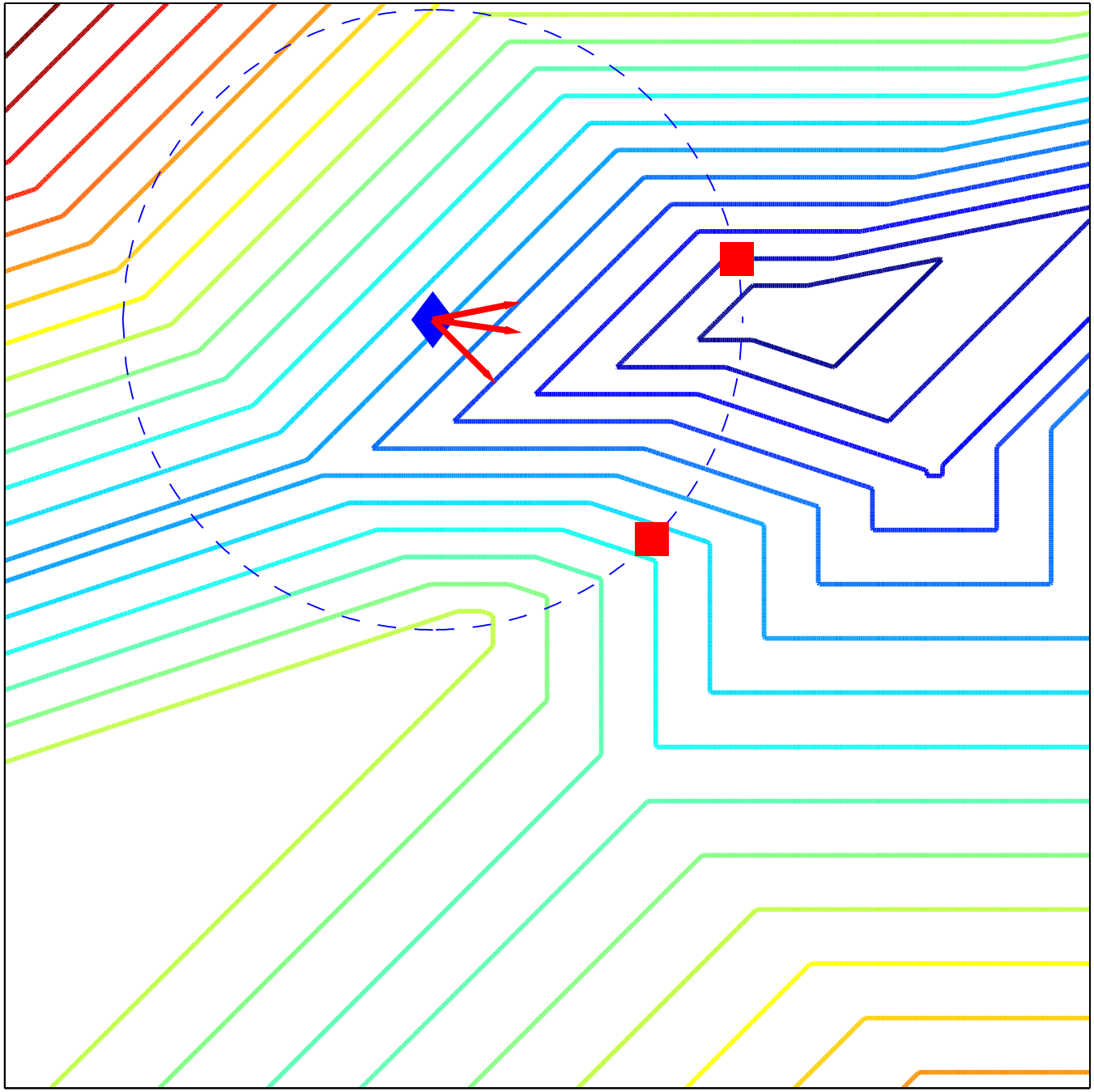


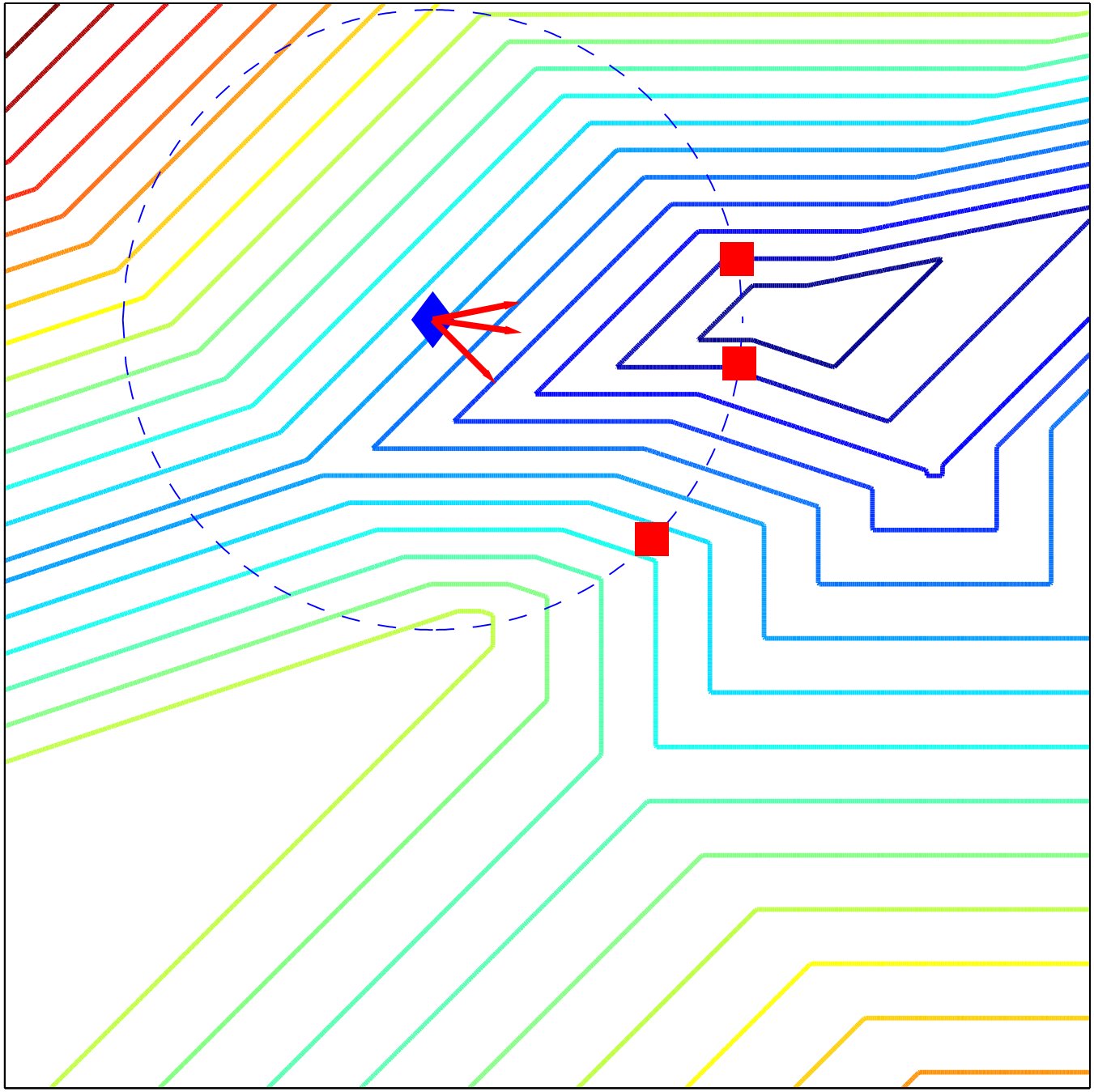


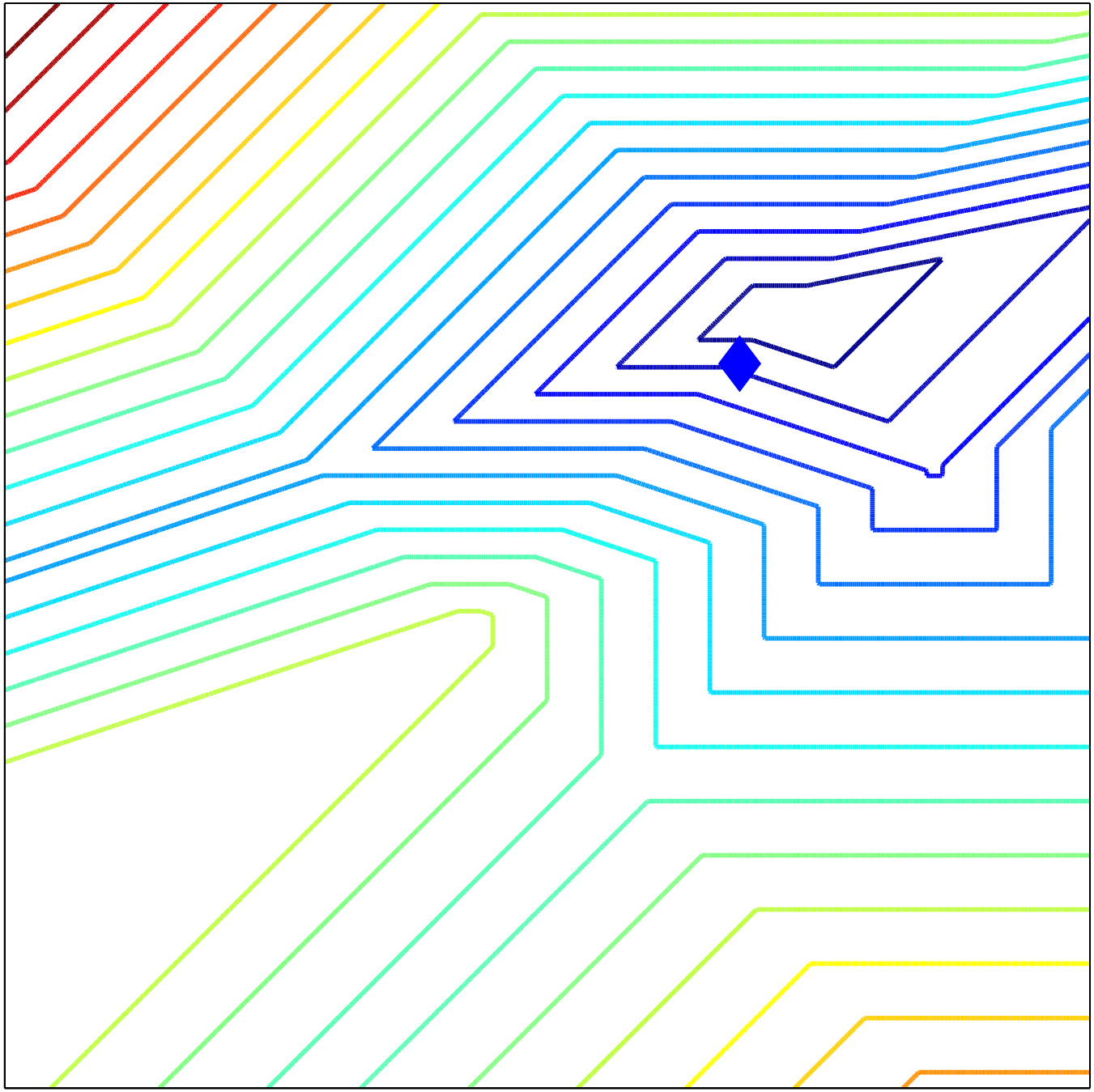


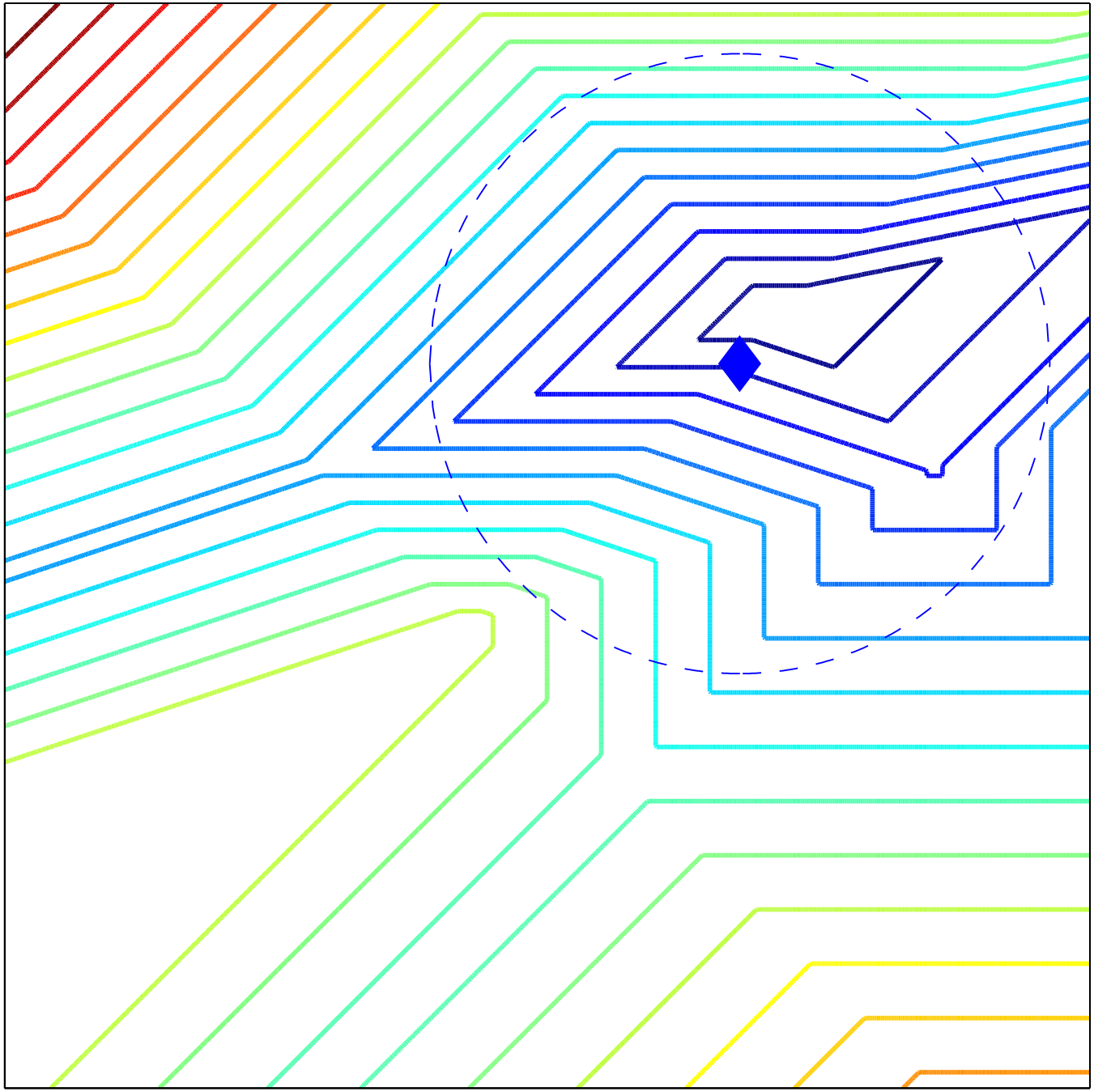


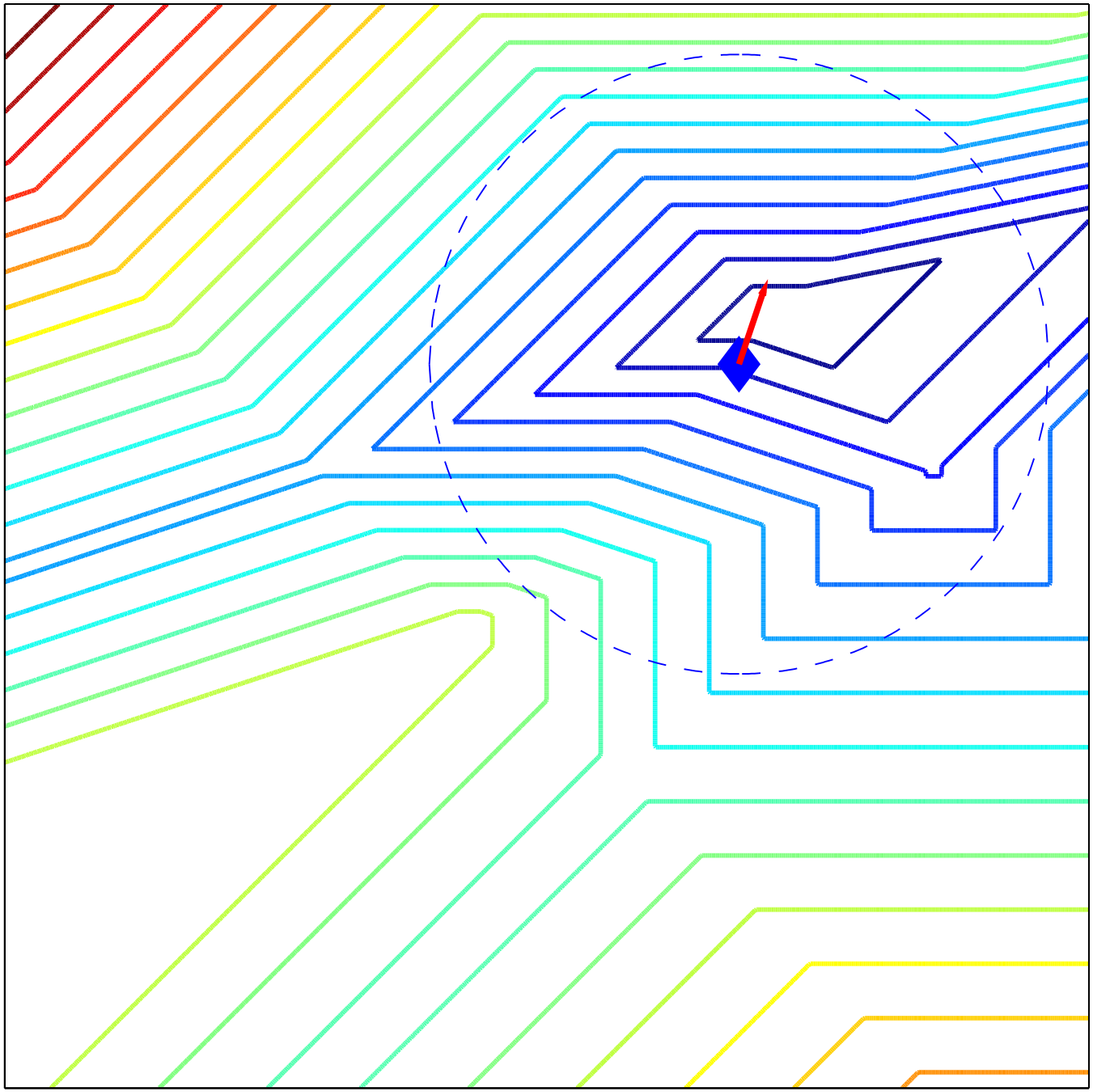


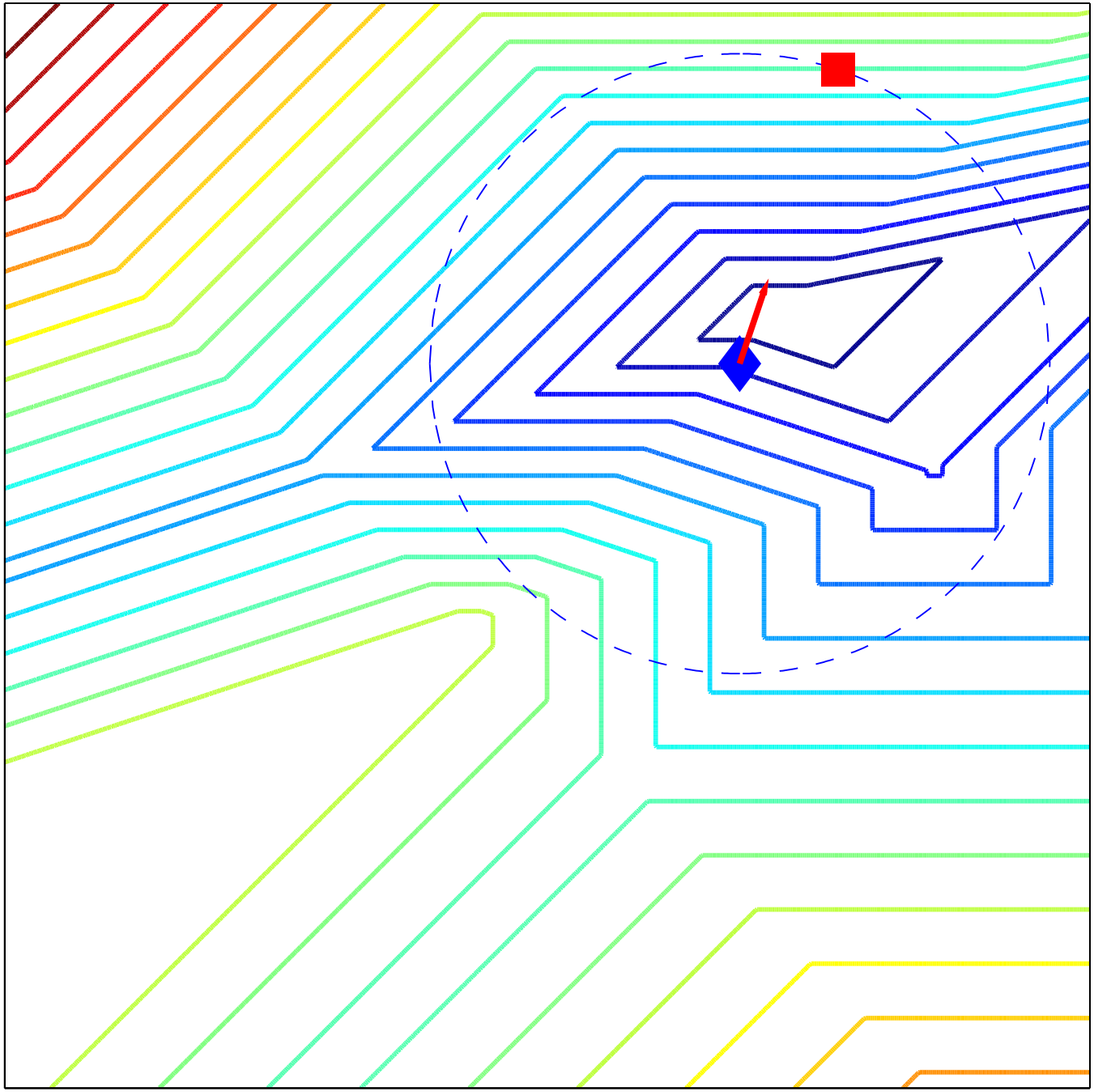


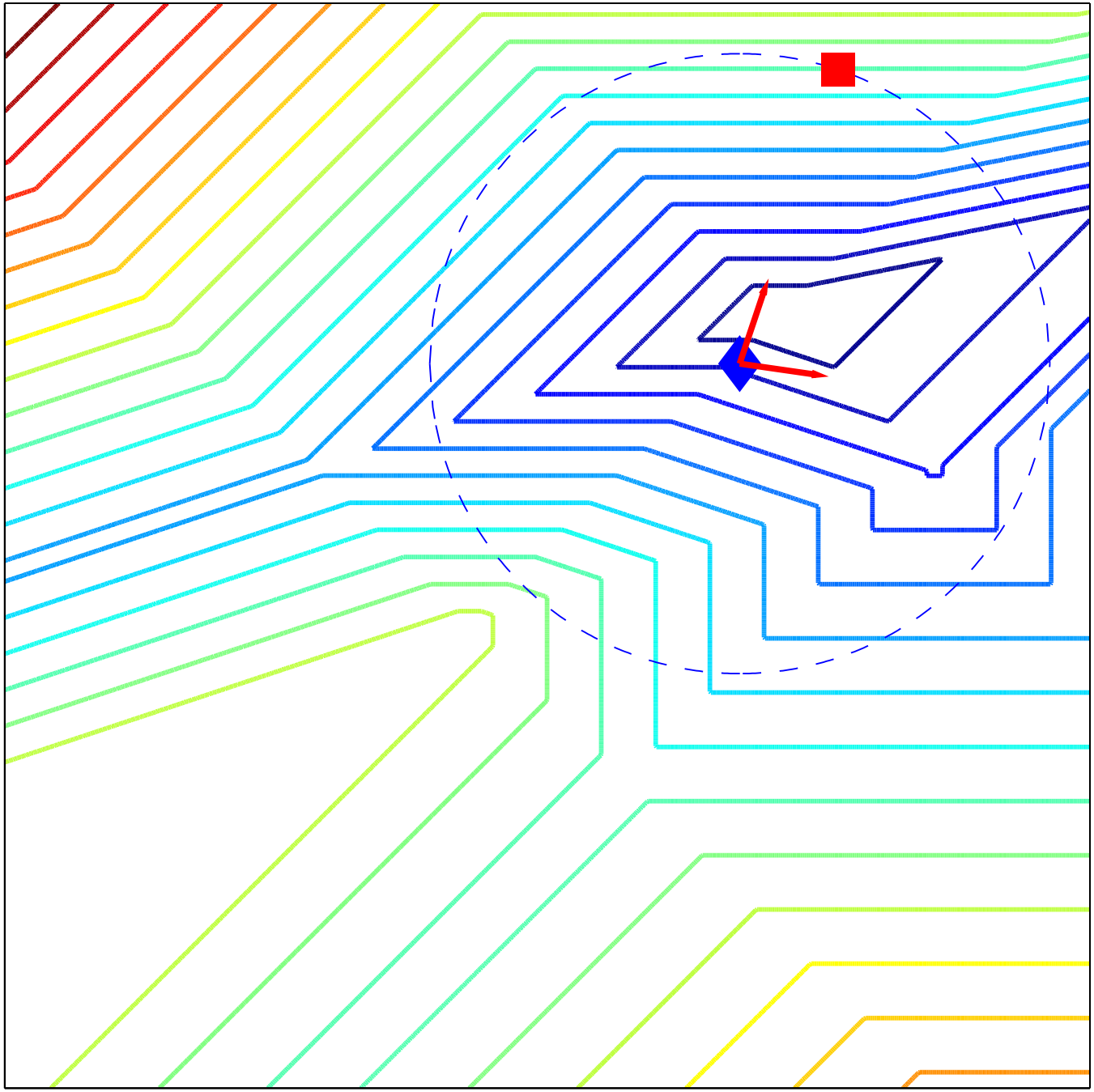


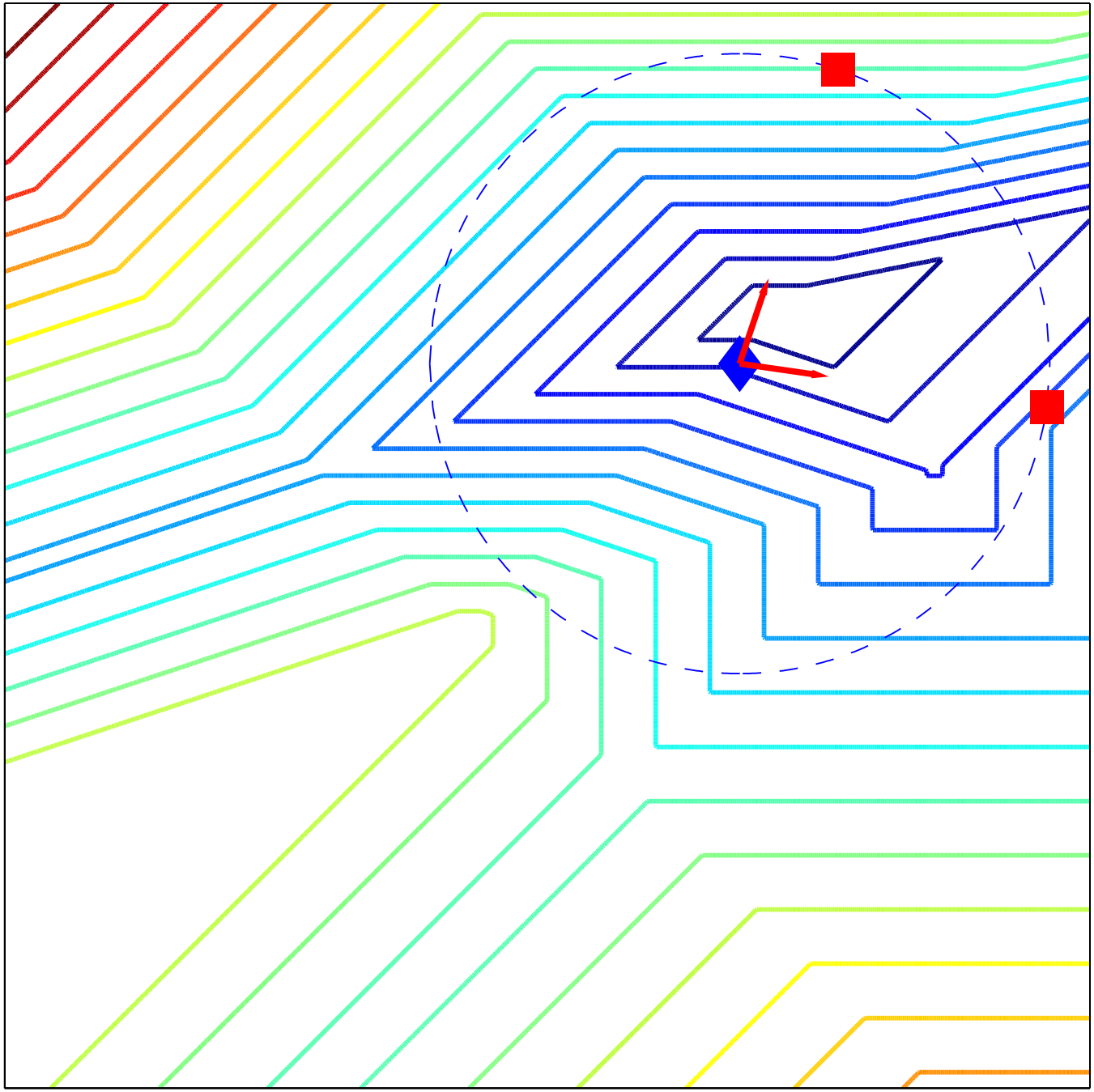


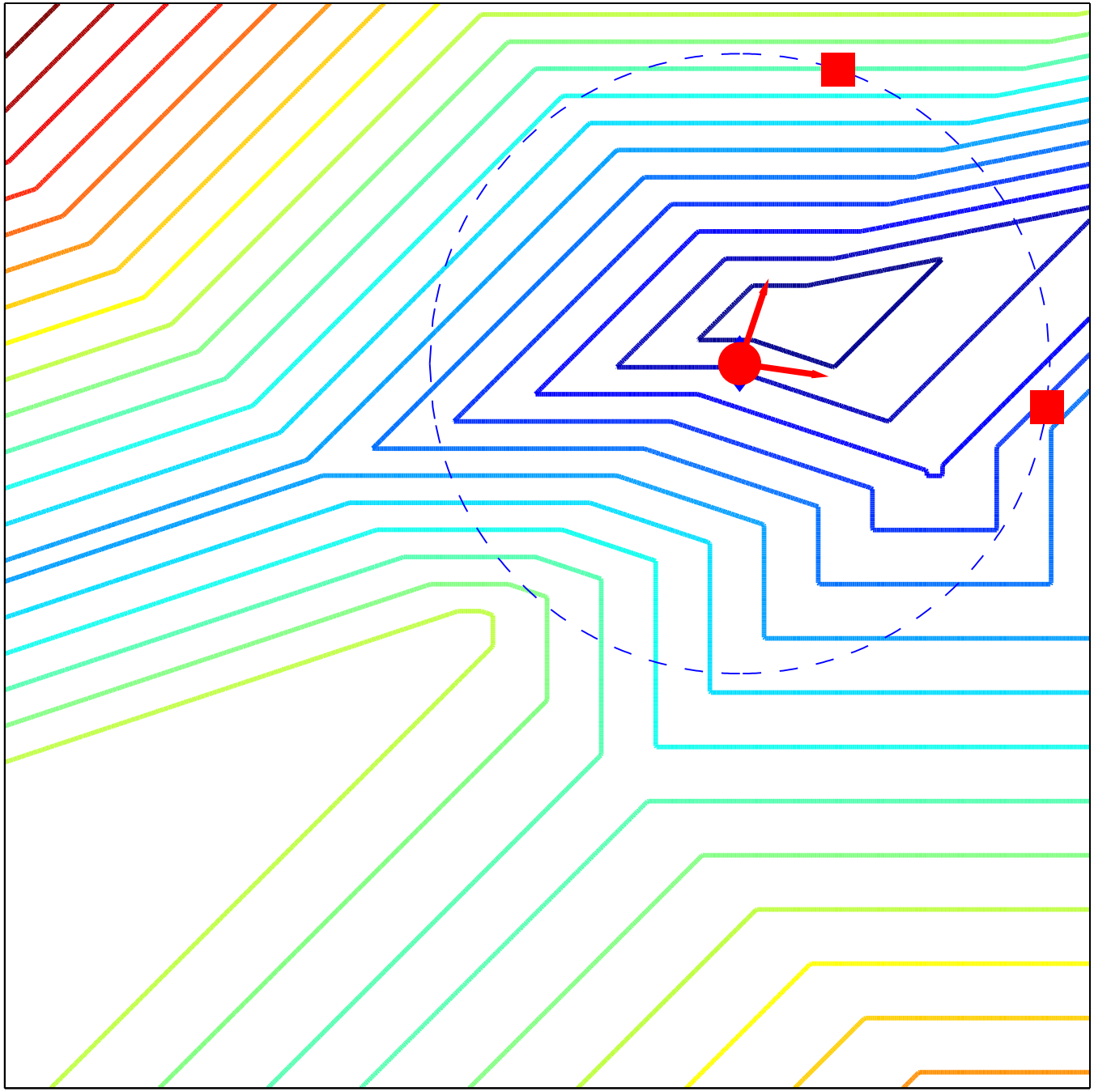


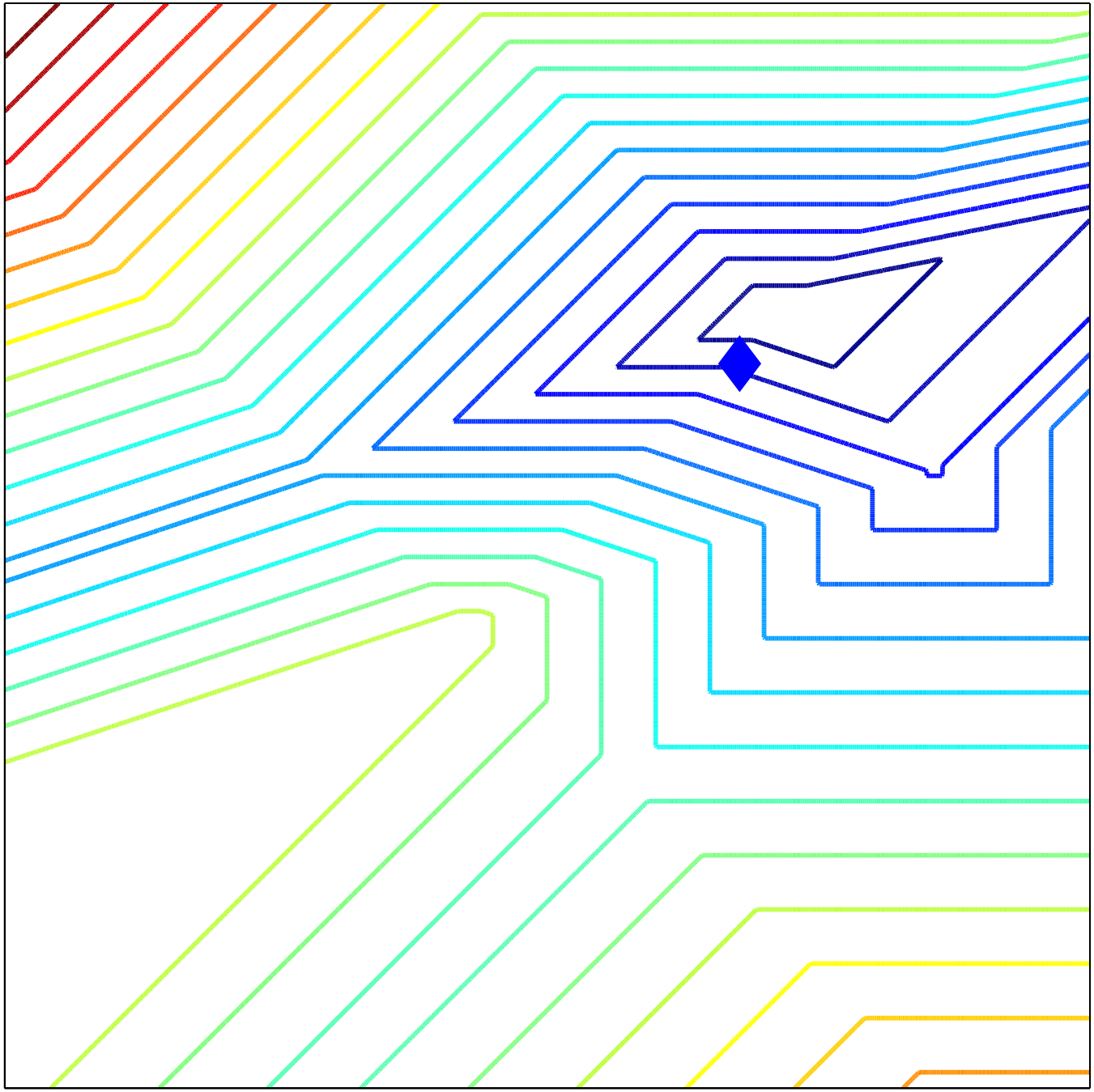


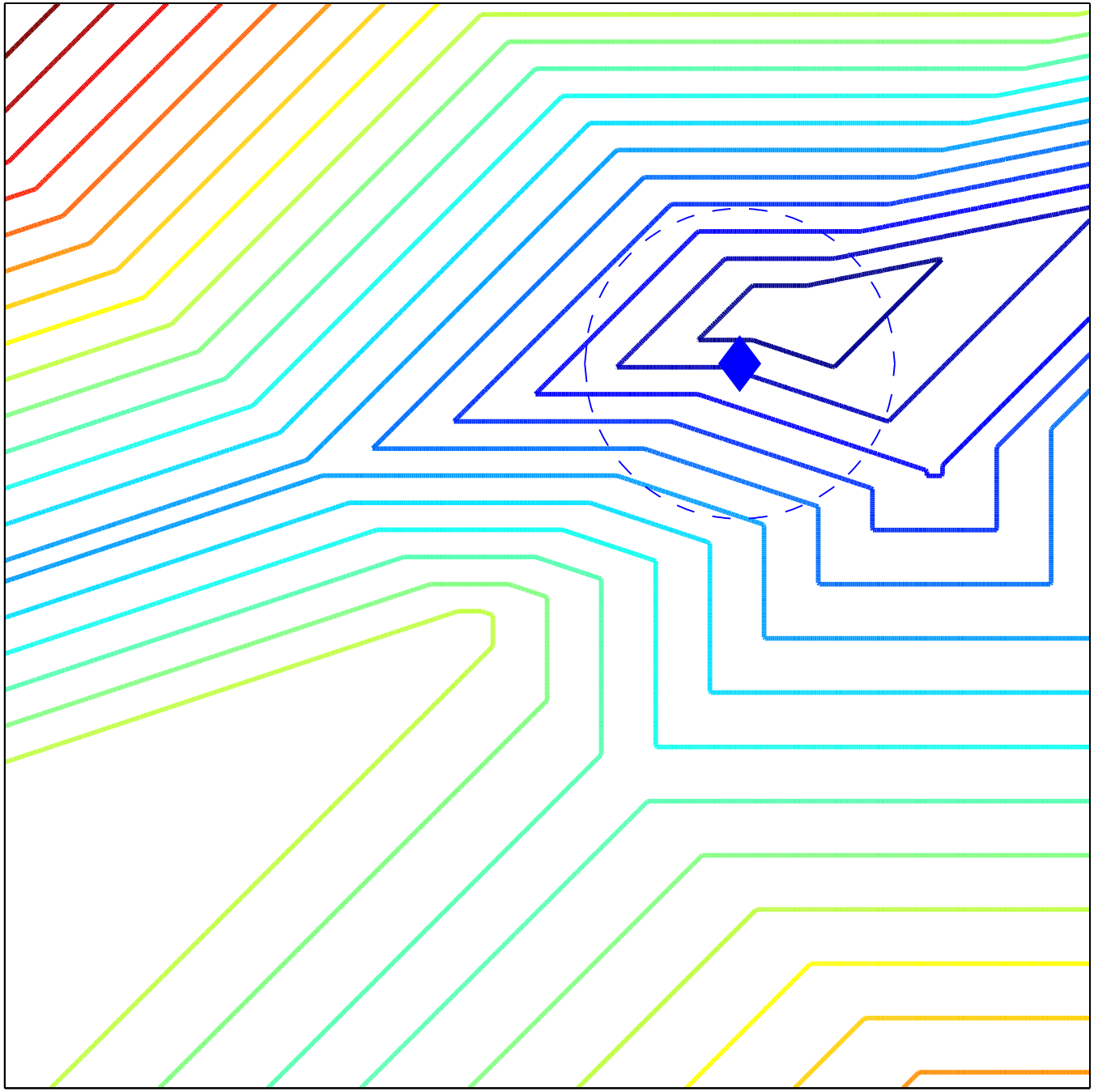


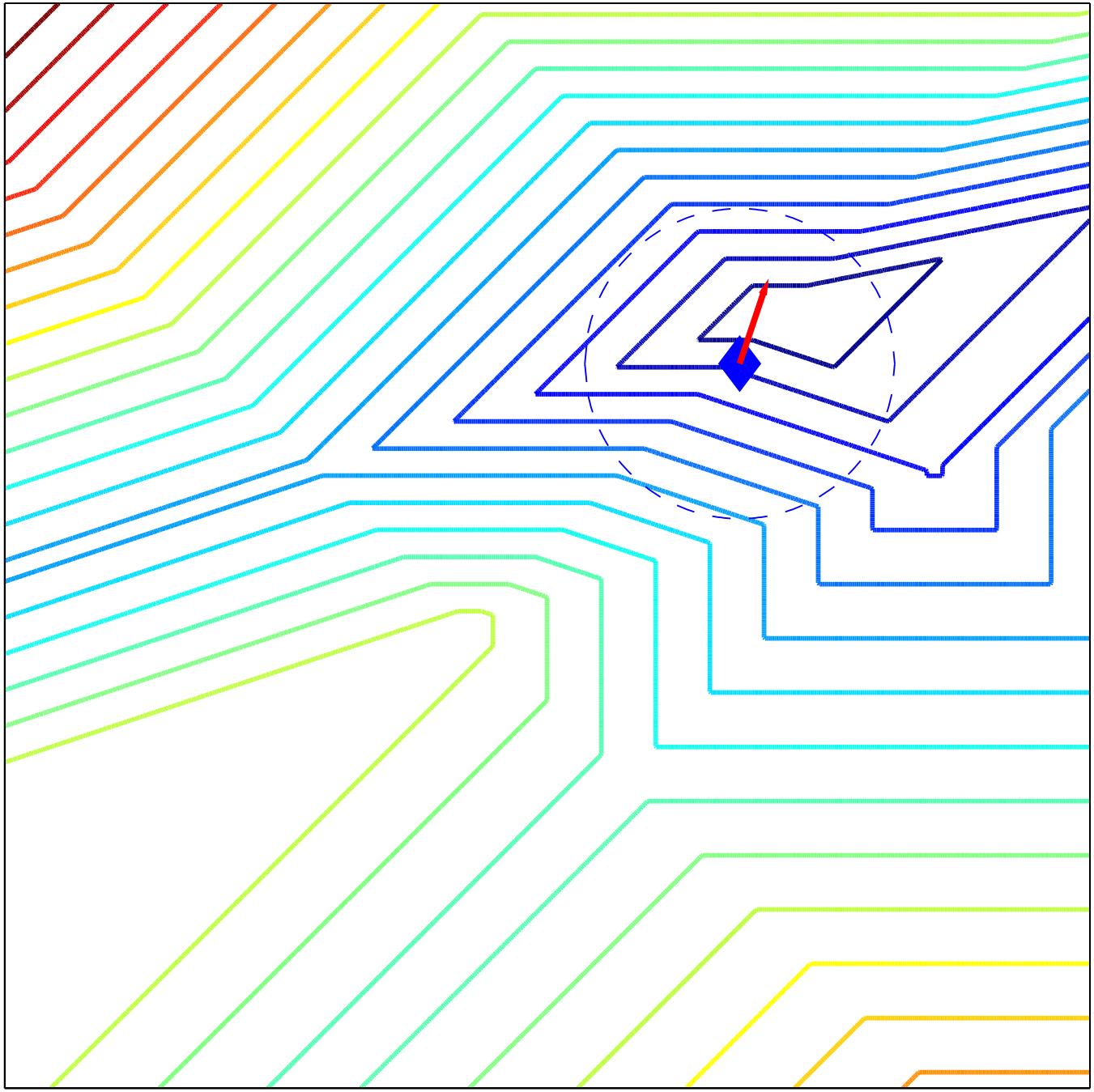


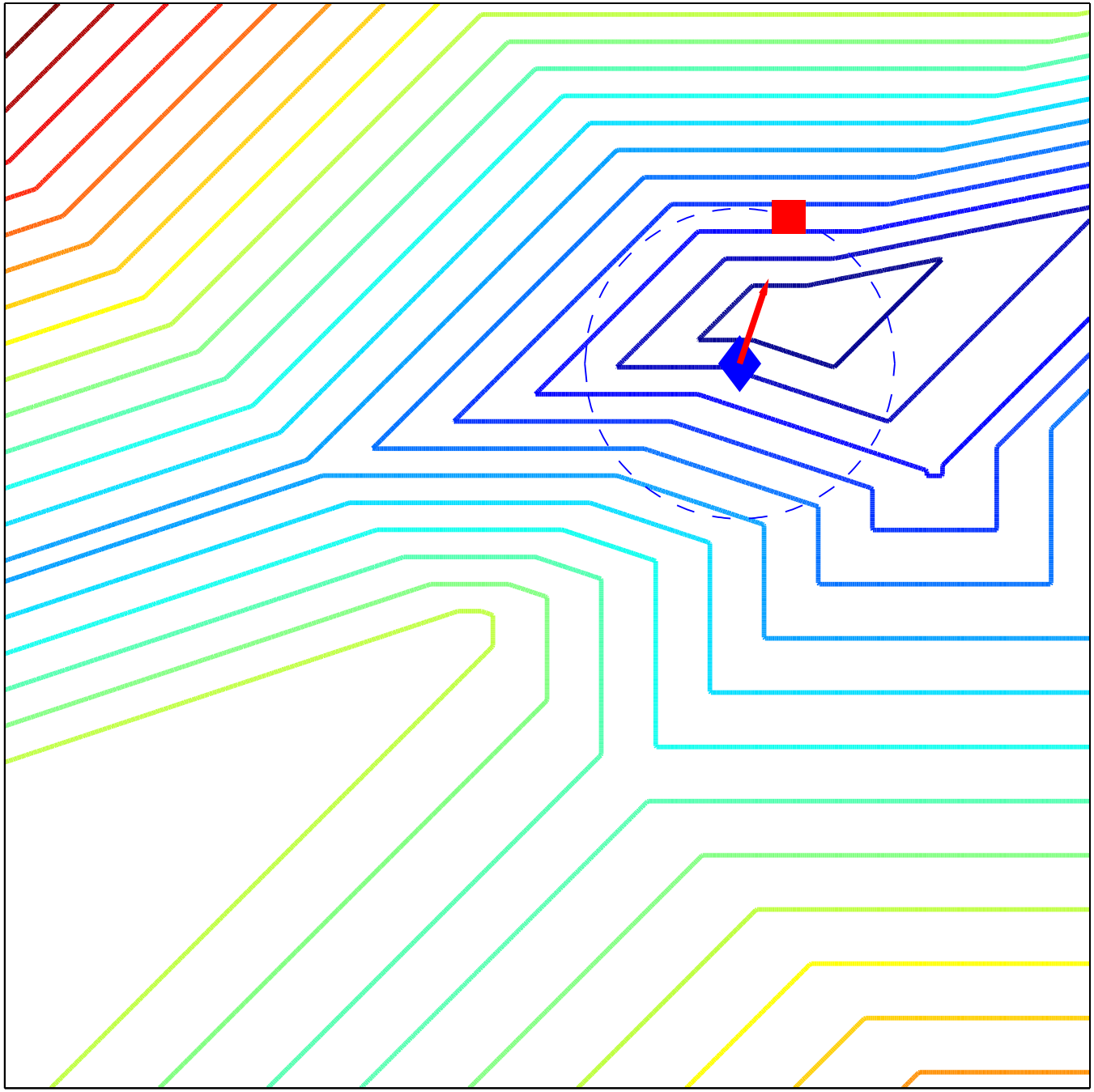


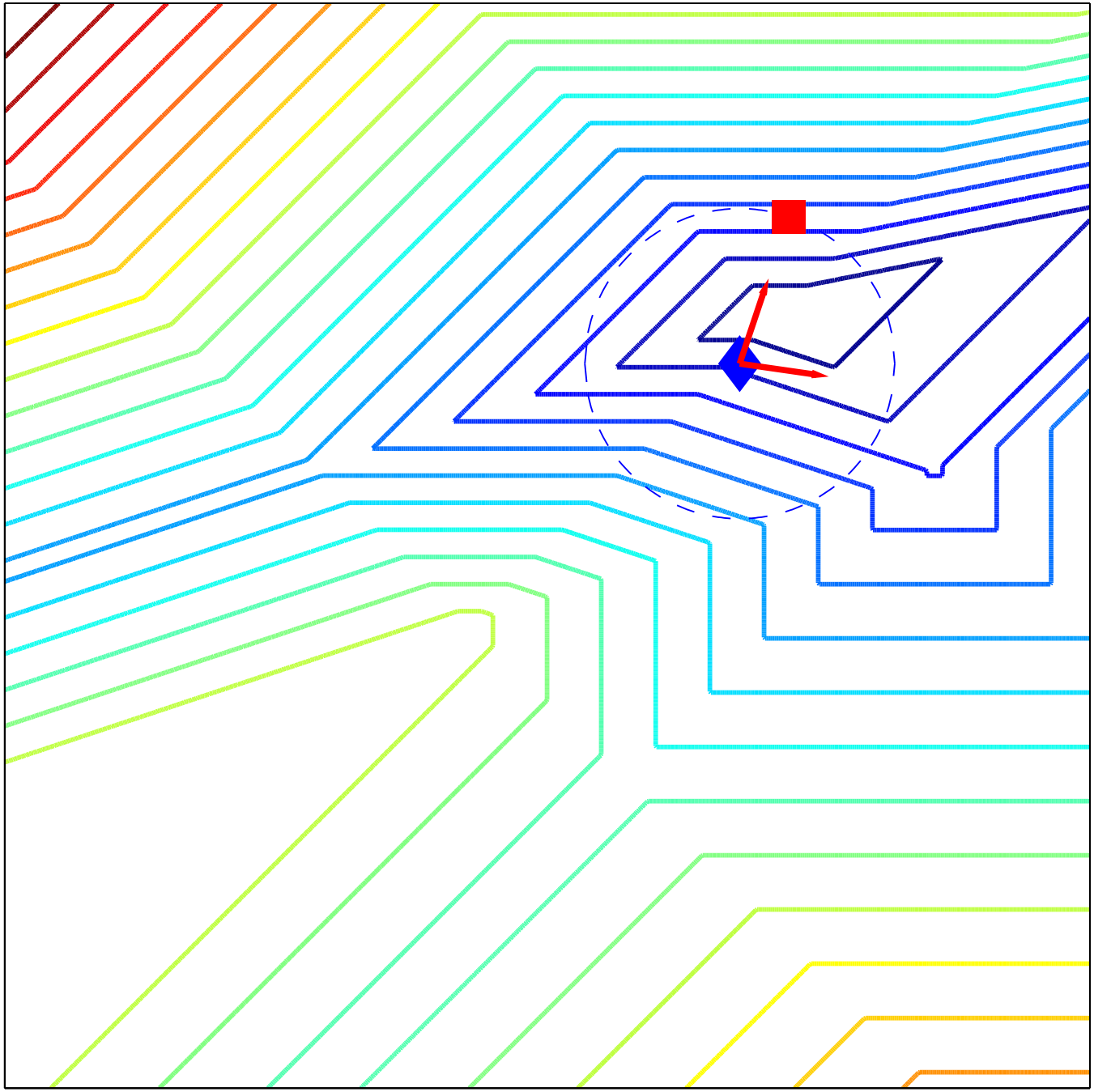


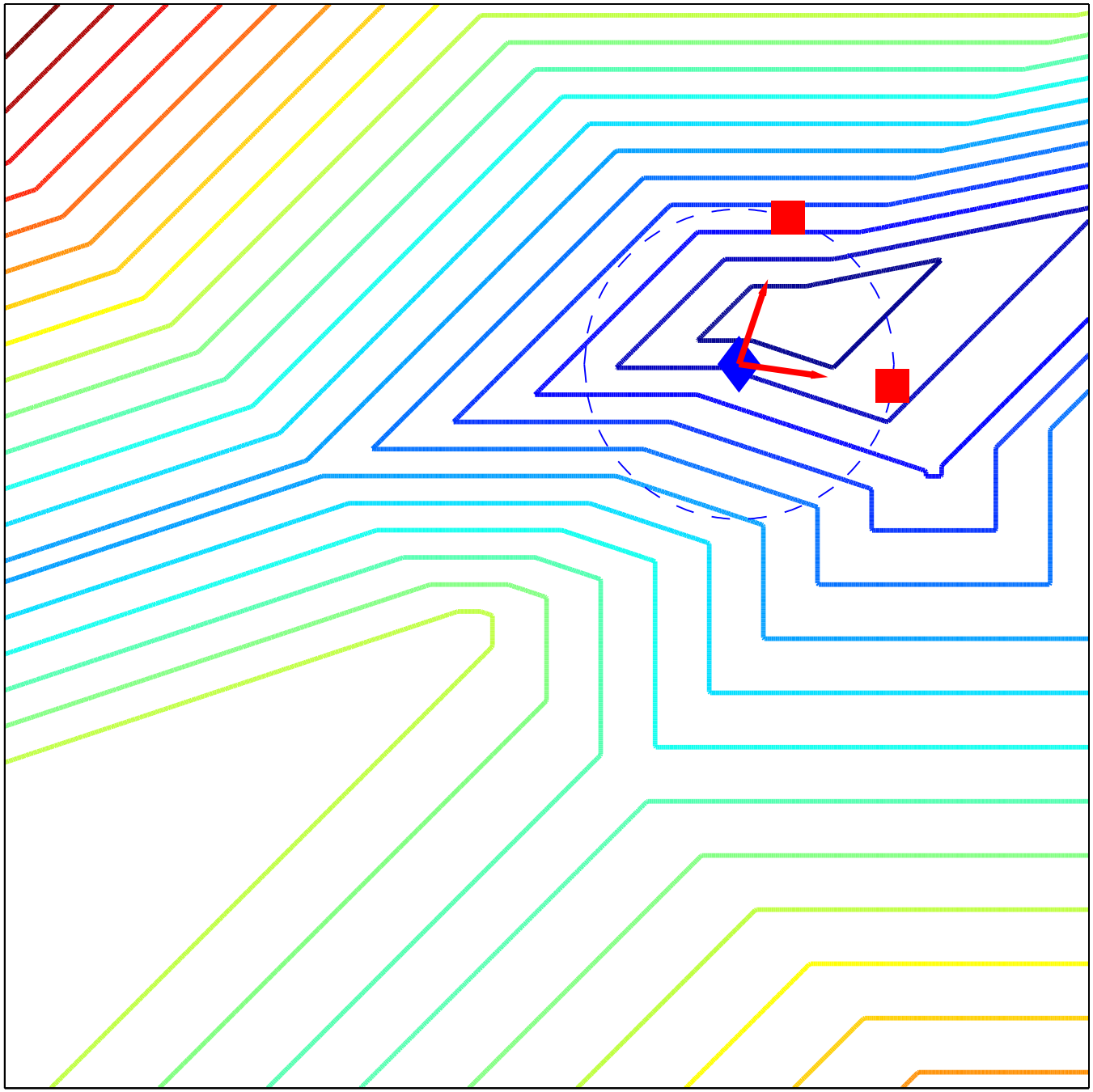


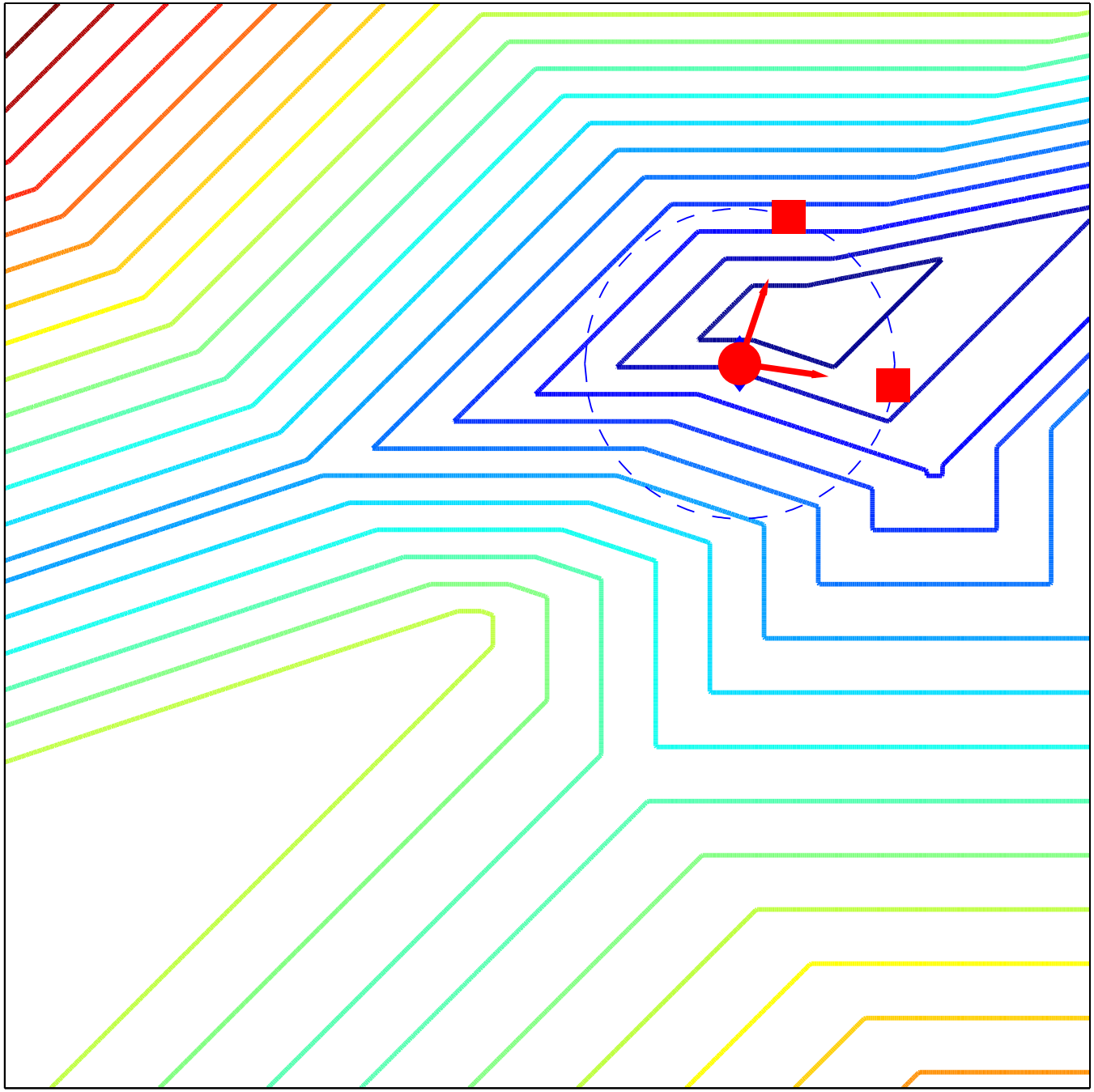


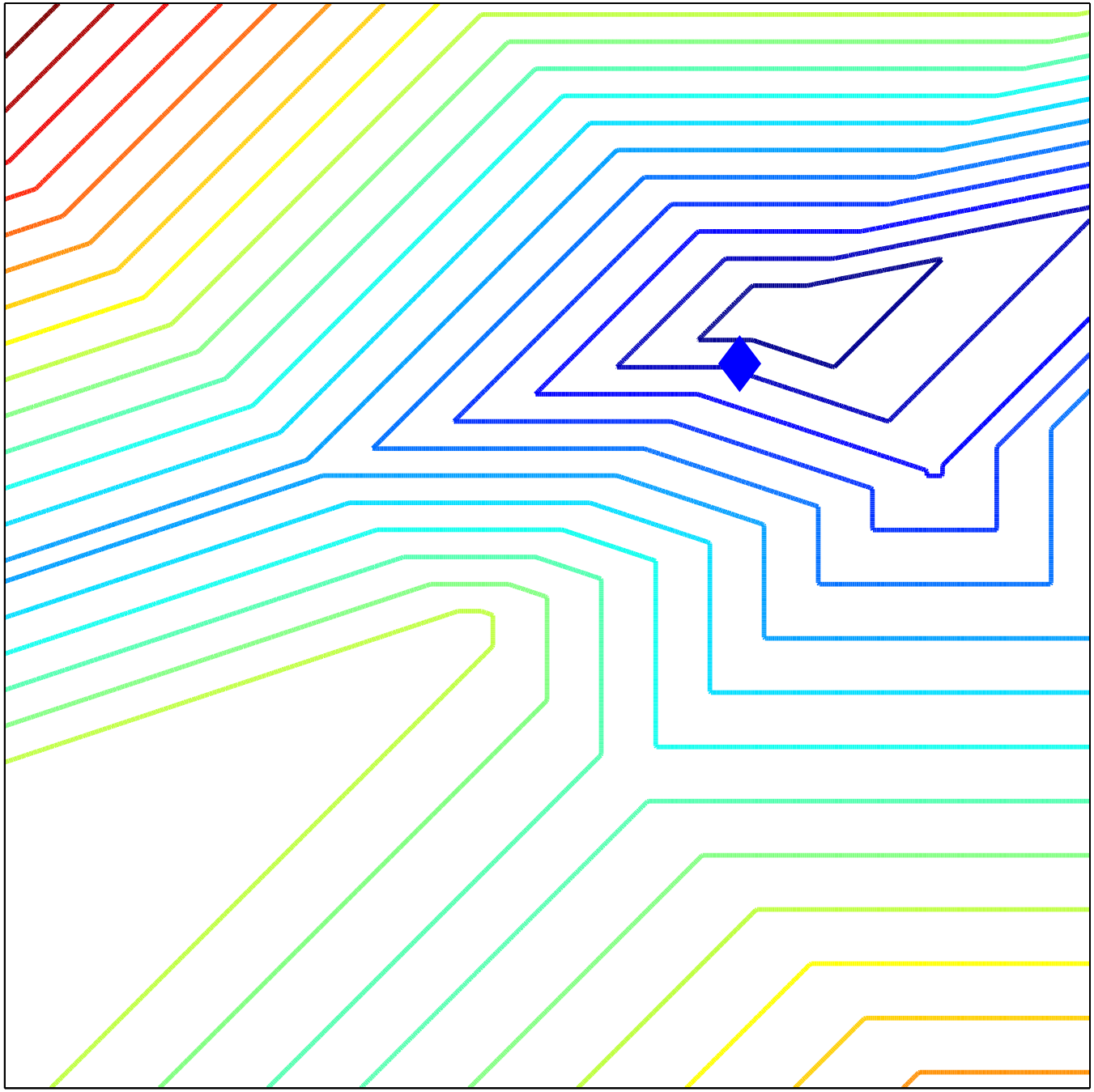


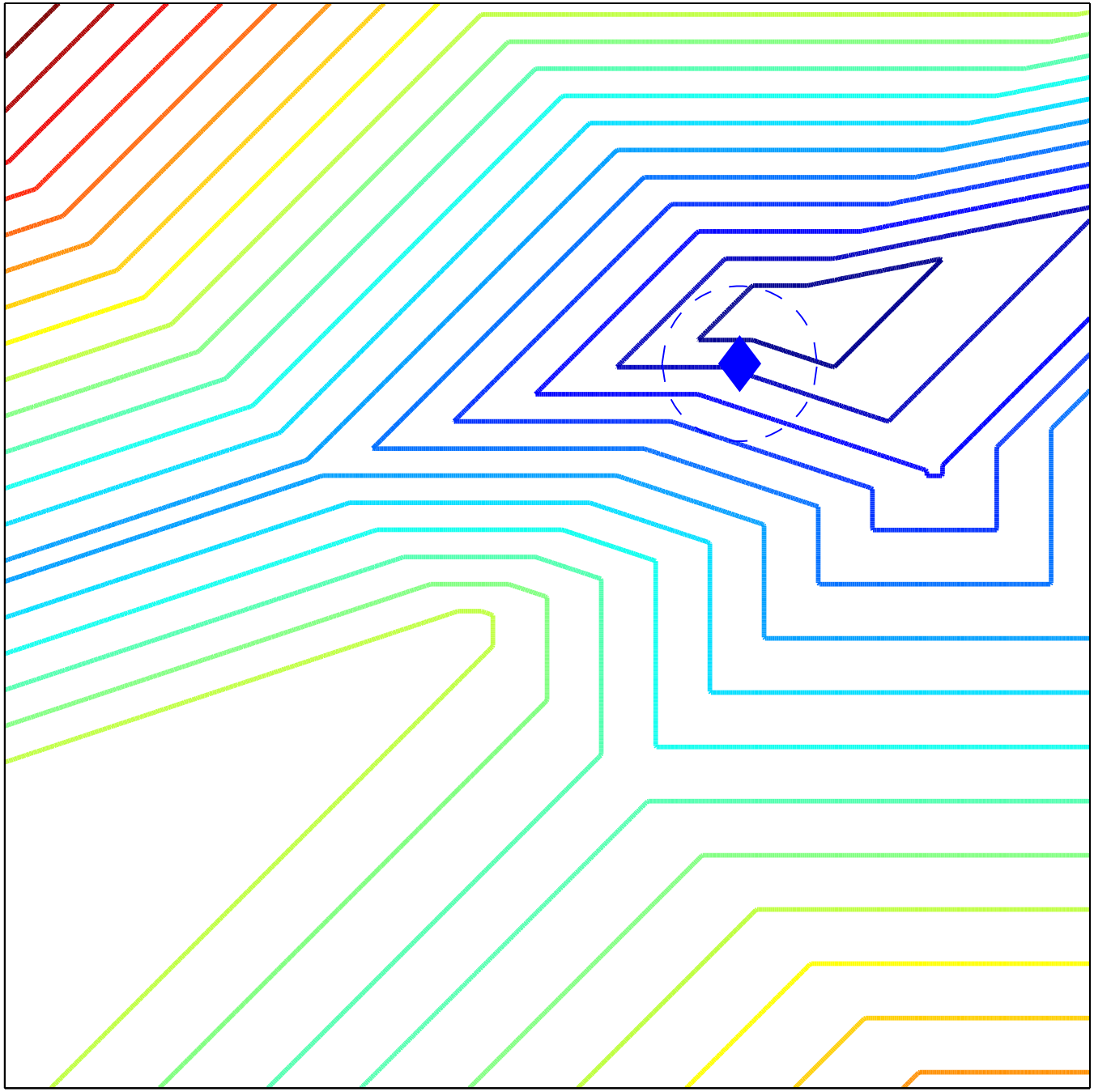


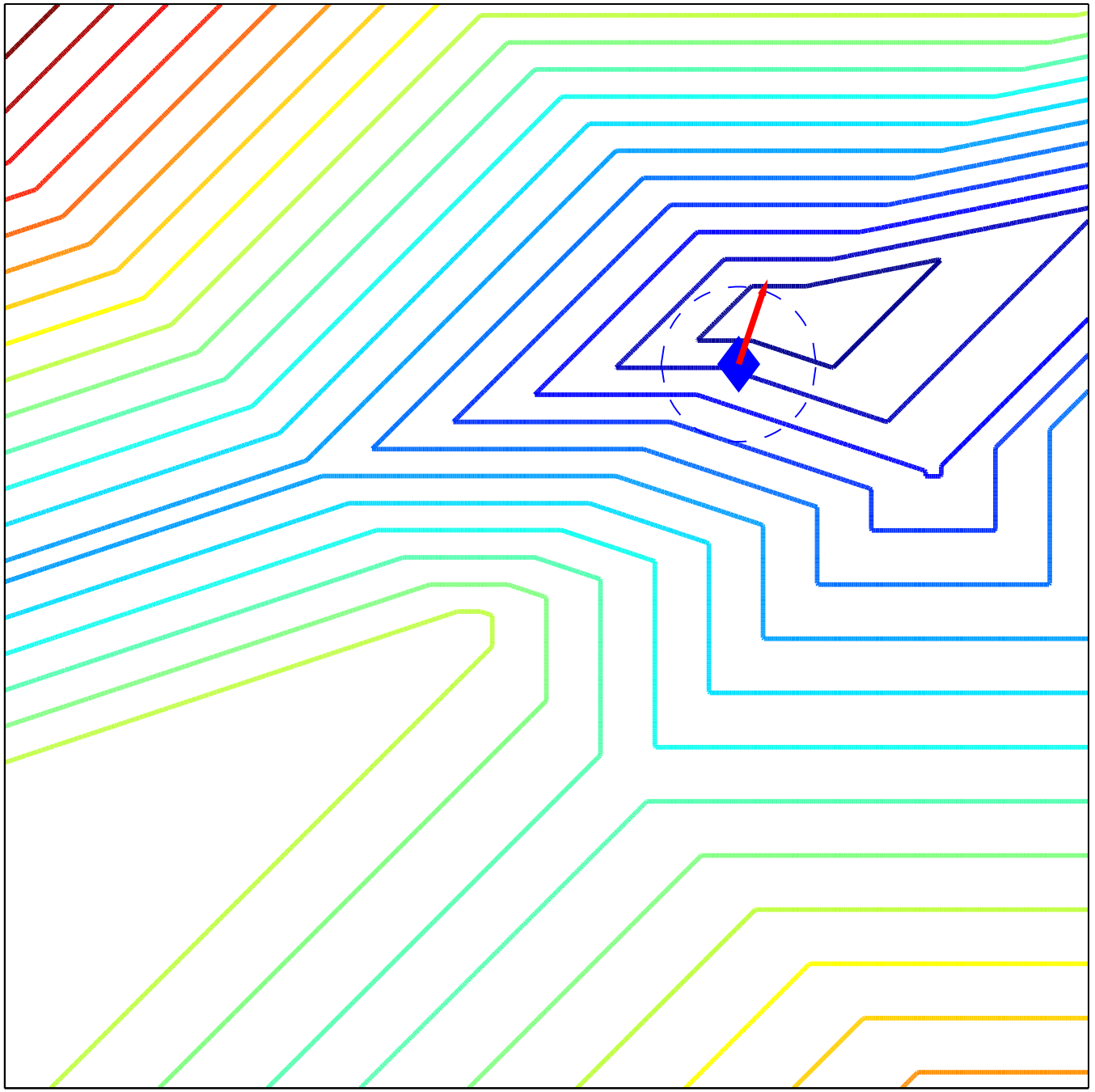


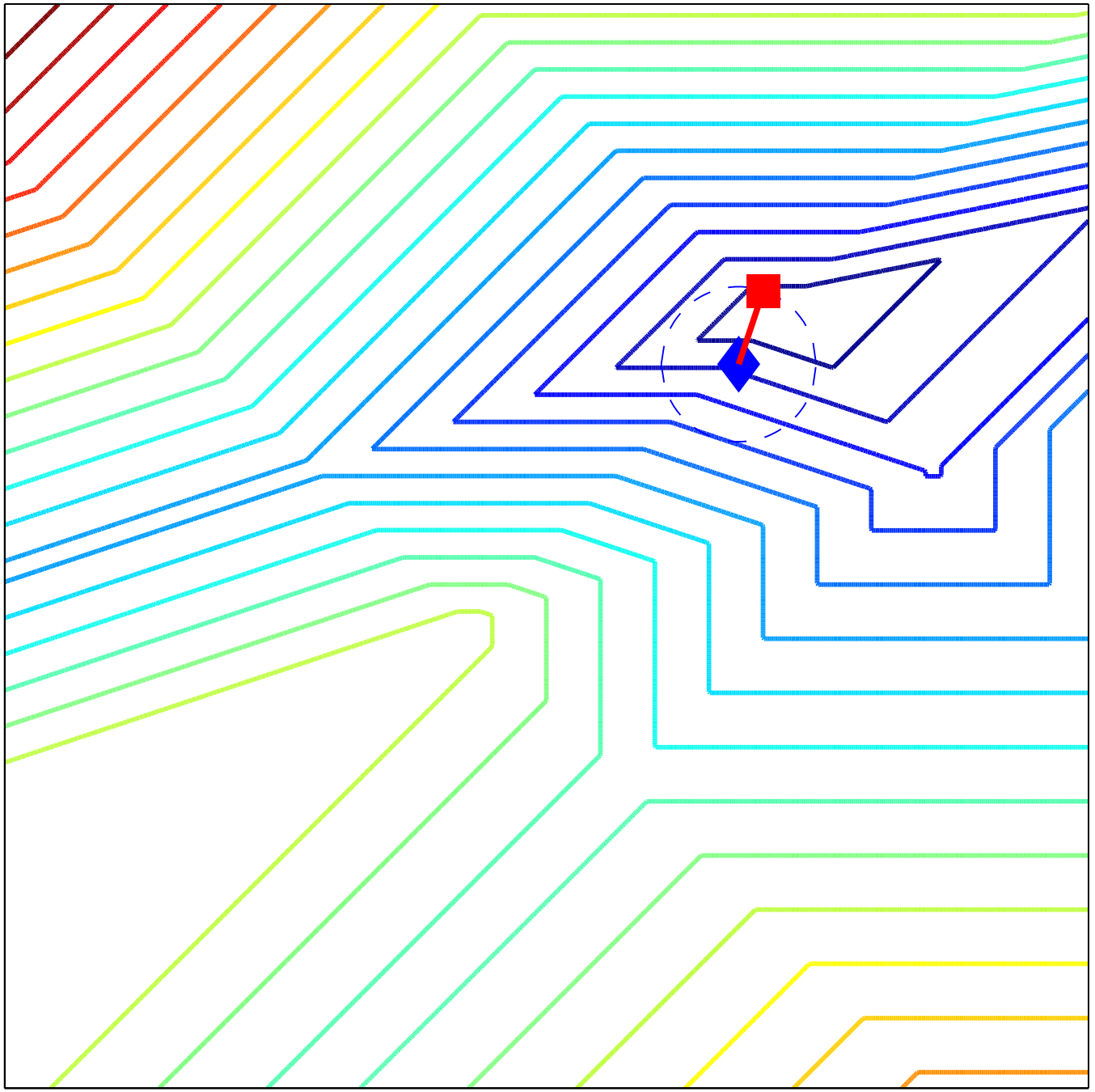


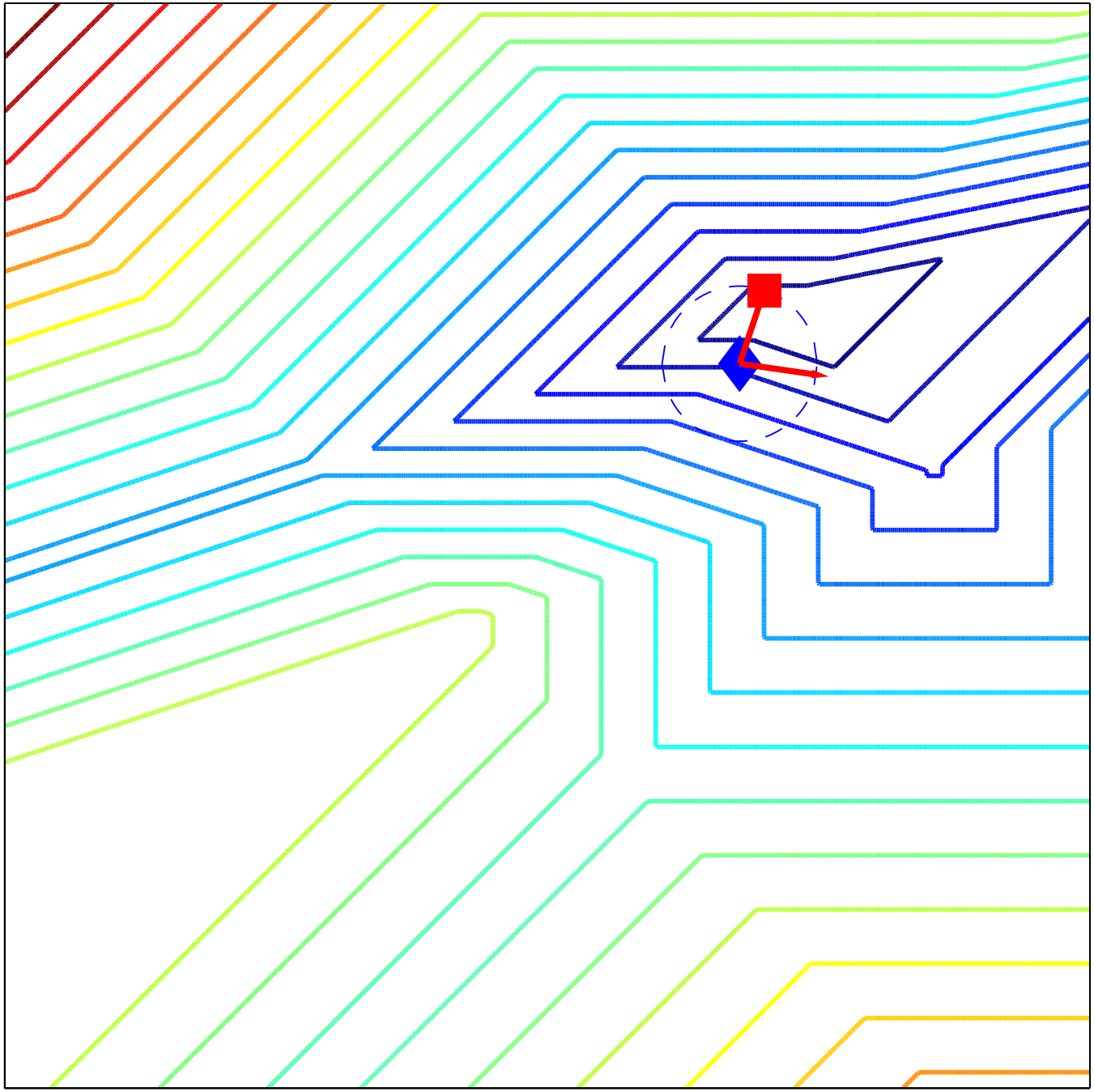


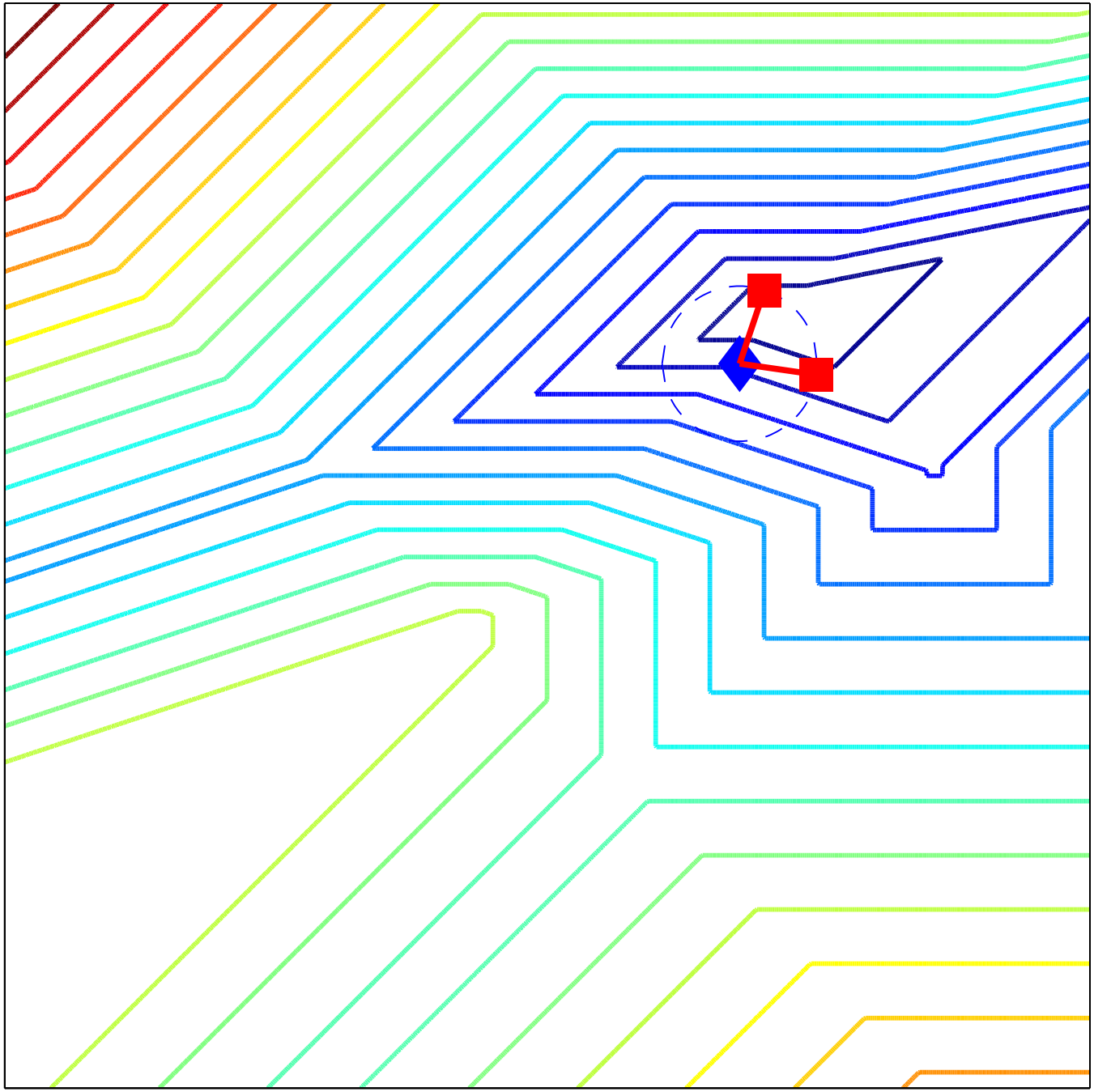


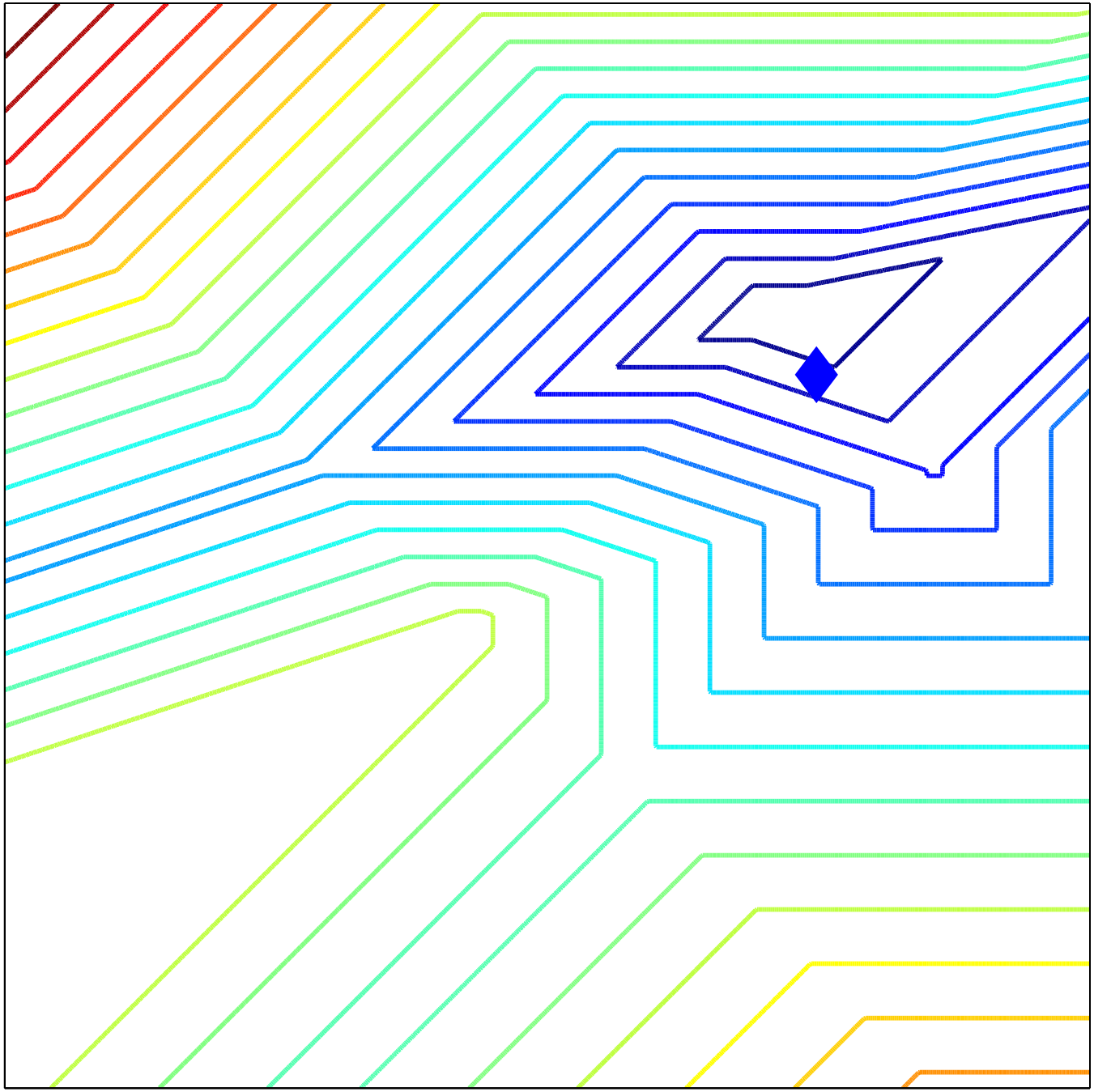












Generator set

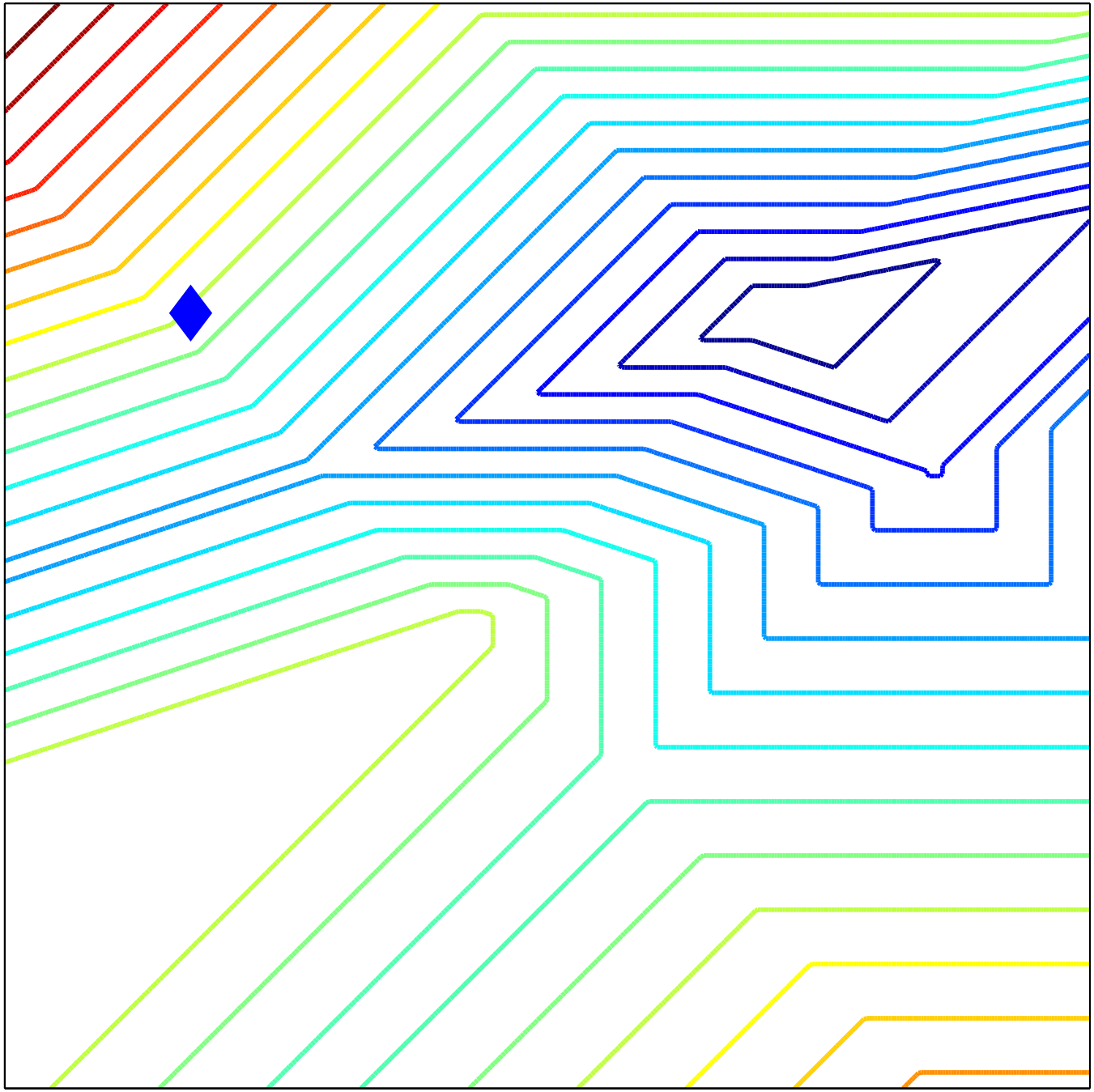
At some iterate x^k,

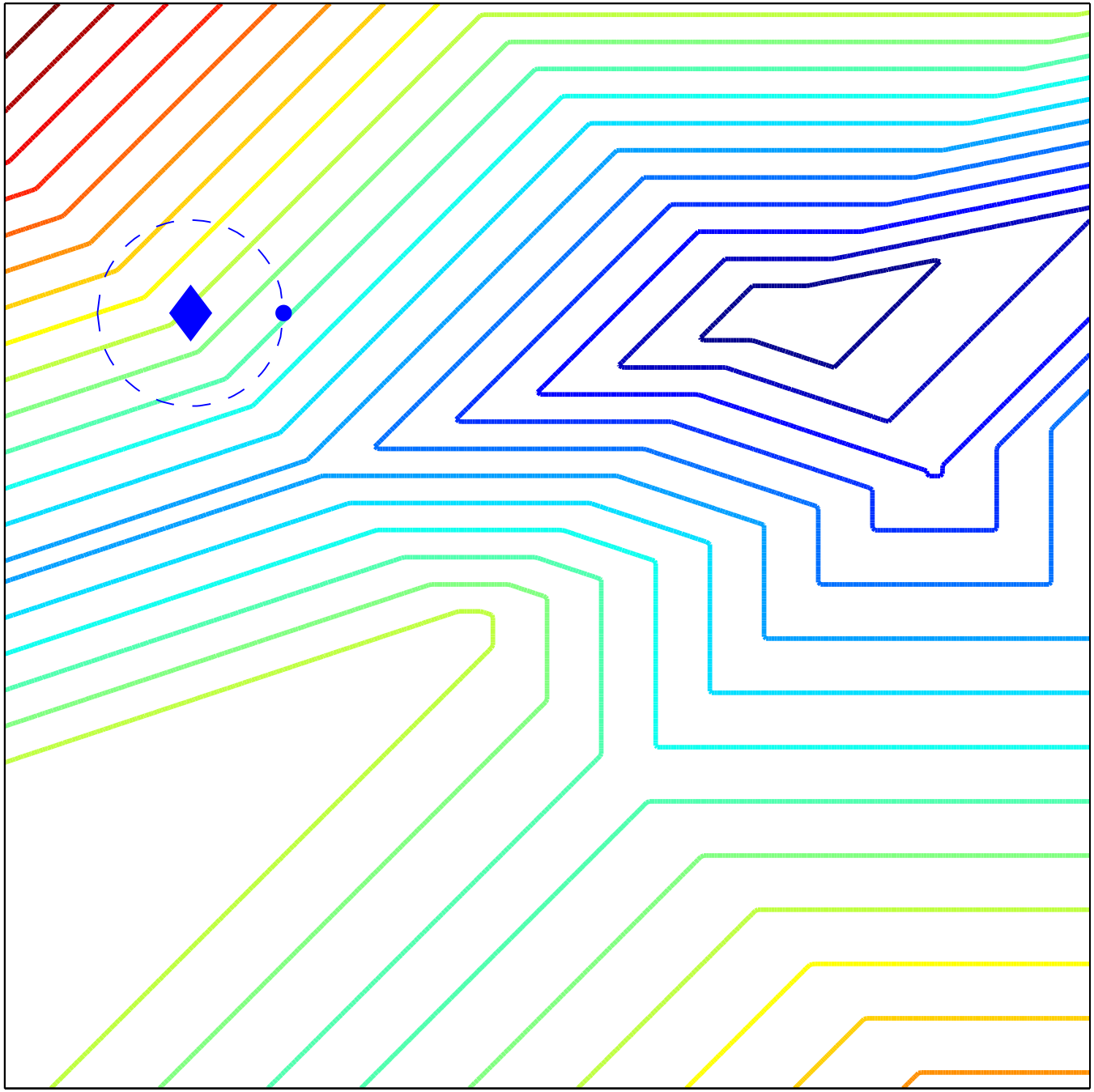
$$\mathfrak{G}^k \triangleq \bigcup_{i \in I_h(F(x^k))} \left\{ \nabla \psi(x^k) + \nabla M(x^k) a_i \right\} \rightarrow \text{MS4PL-1}$$

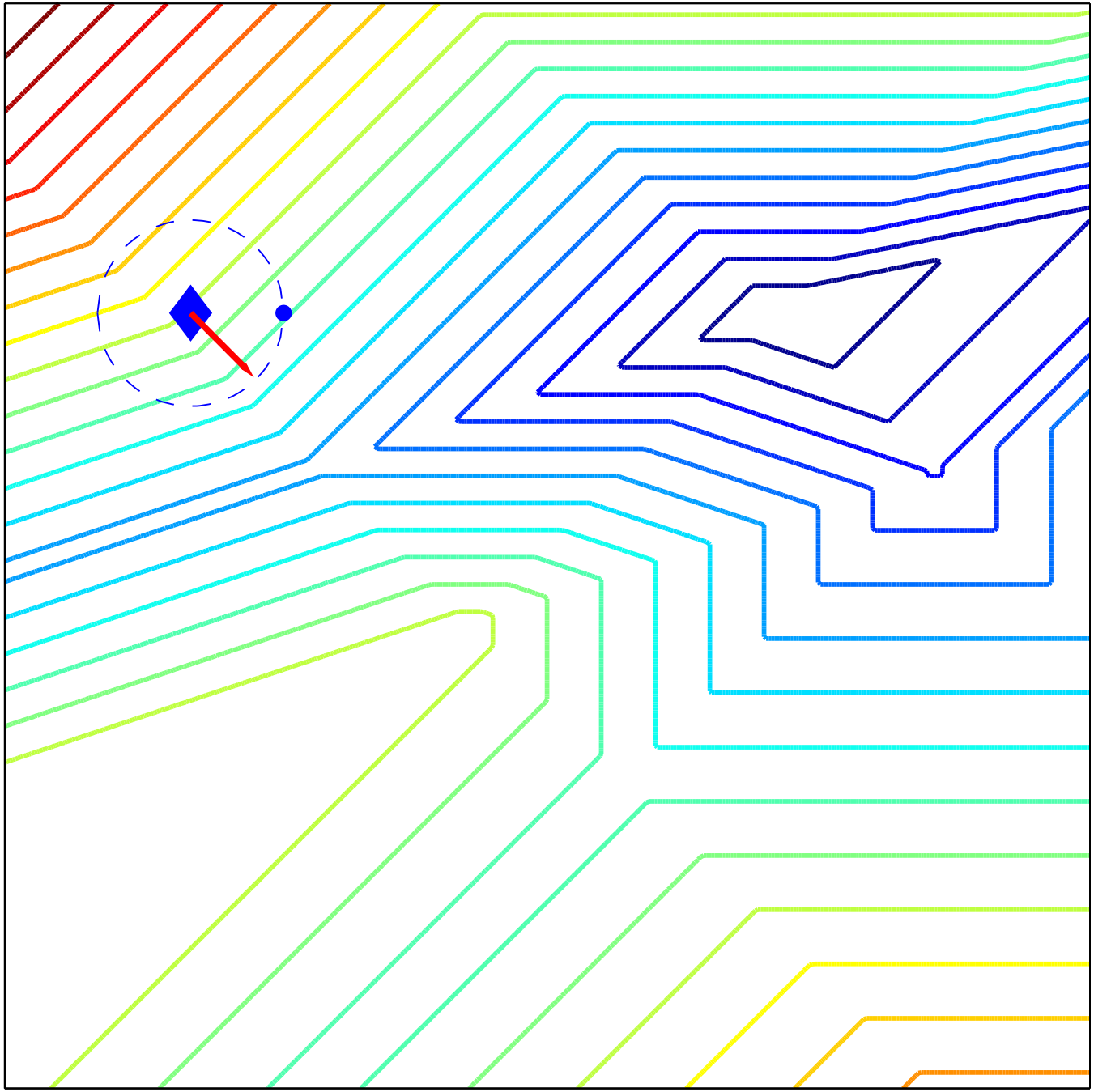
where $I_h(F(x^k))$ is the set of essentially active indices of h at $F(x^k)$.

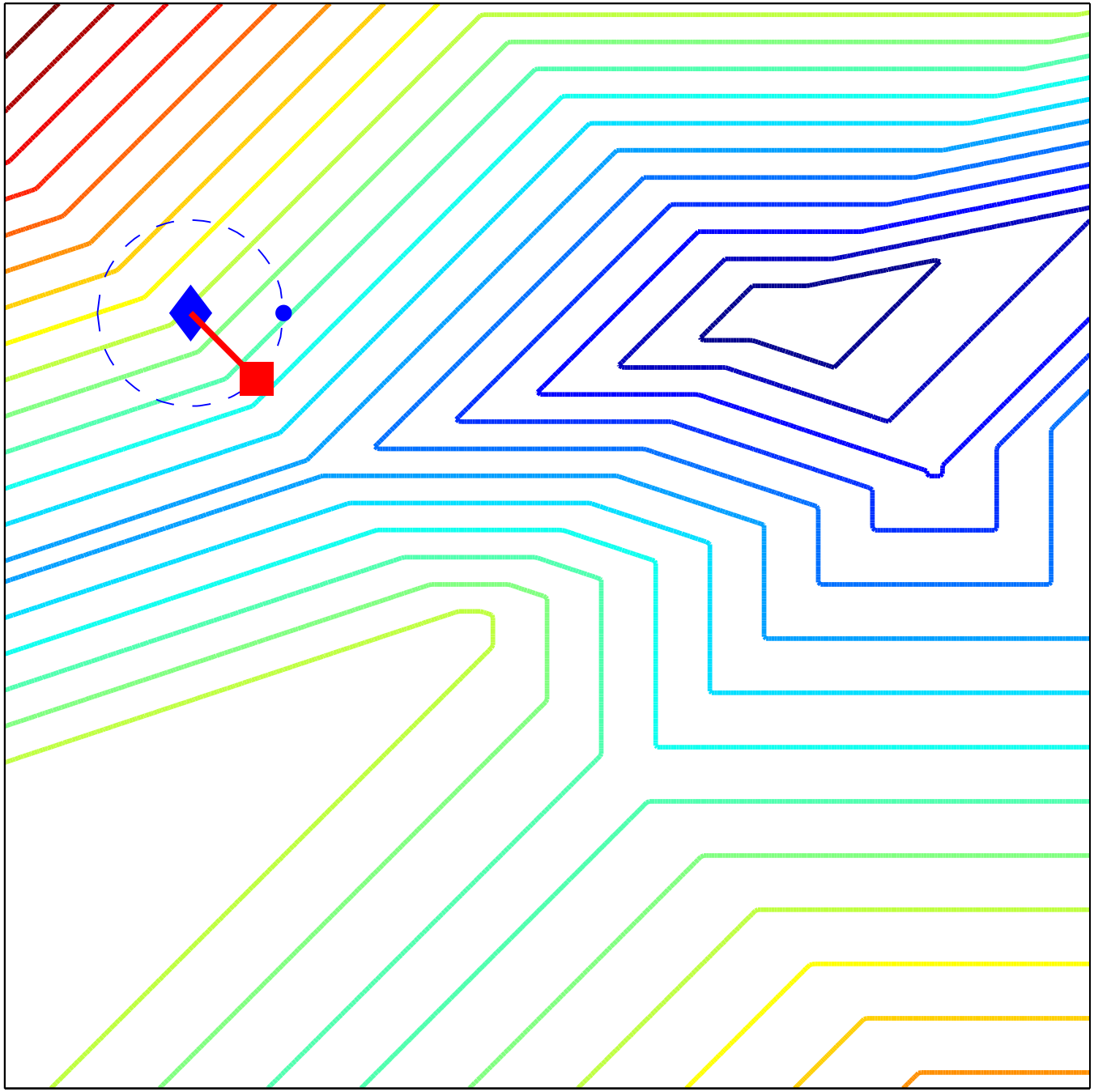
Or, given a set of points $Y = \left\{ x^k, y^2, \ldots, y^p \right\} \subset \mathcal{B}(x^k, \Delta_k)$,

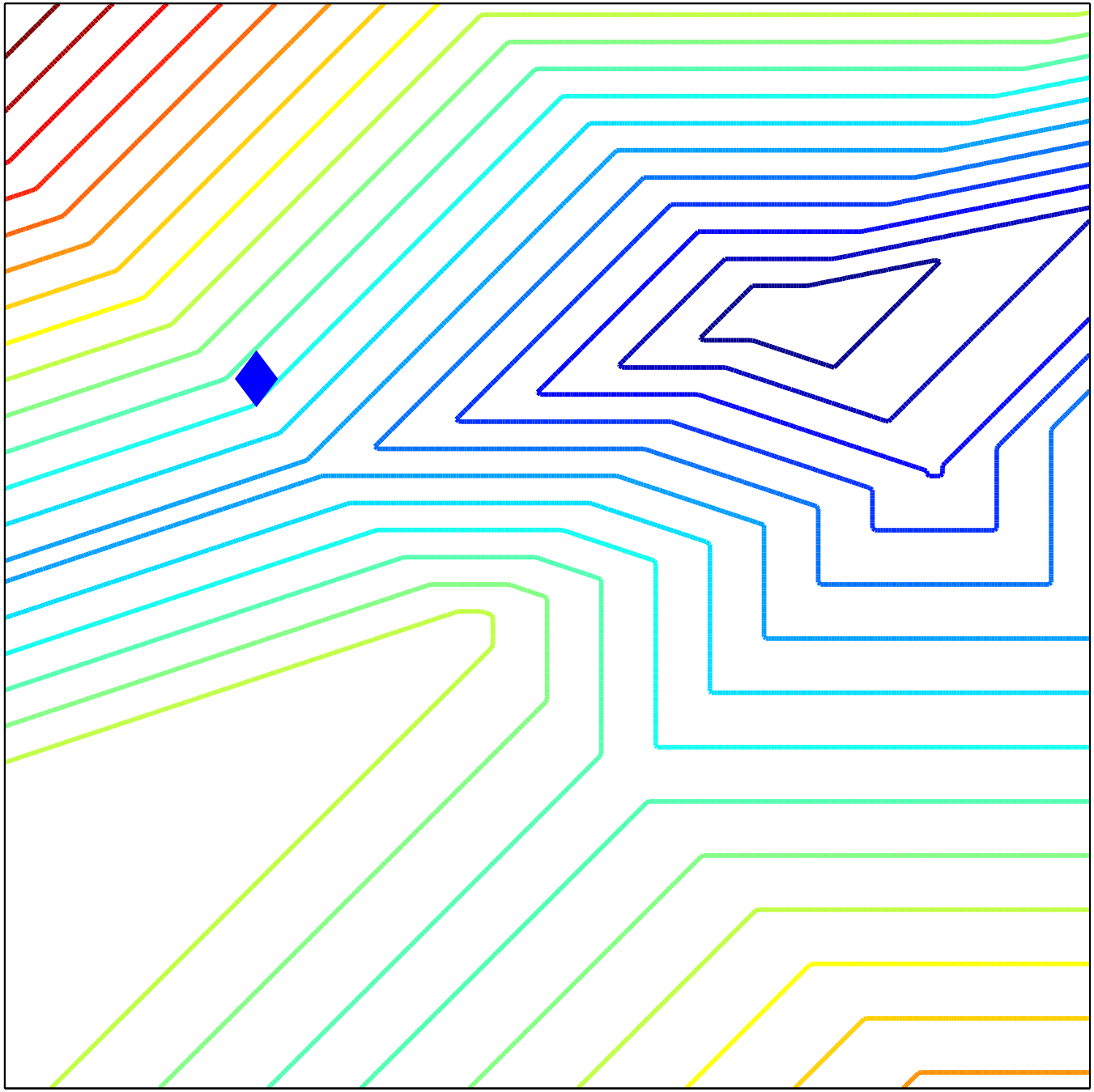
$$\mathfrak{G}^k \triangleq \bigcup_{y \in Y} \bigcup_{i \in I_h(F(y))} \left\{ \nabla \psi(x^k) + \nabla M(x^k) a_i \right\} \rightarrow \text{MS4PL-2}$$

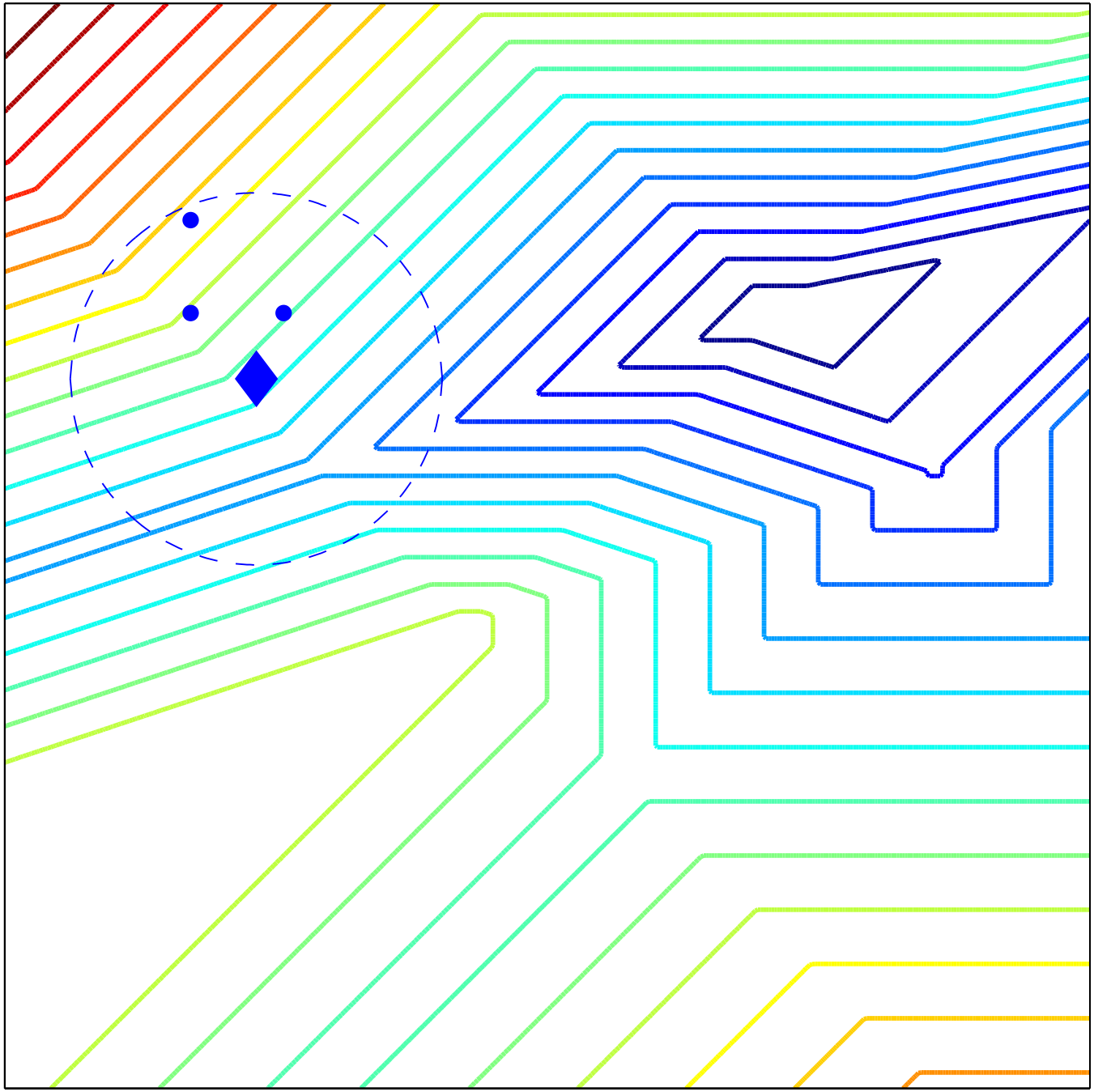


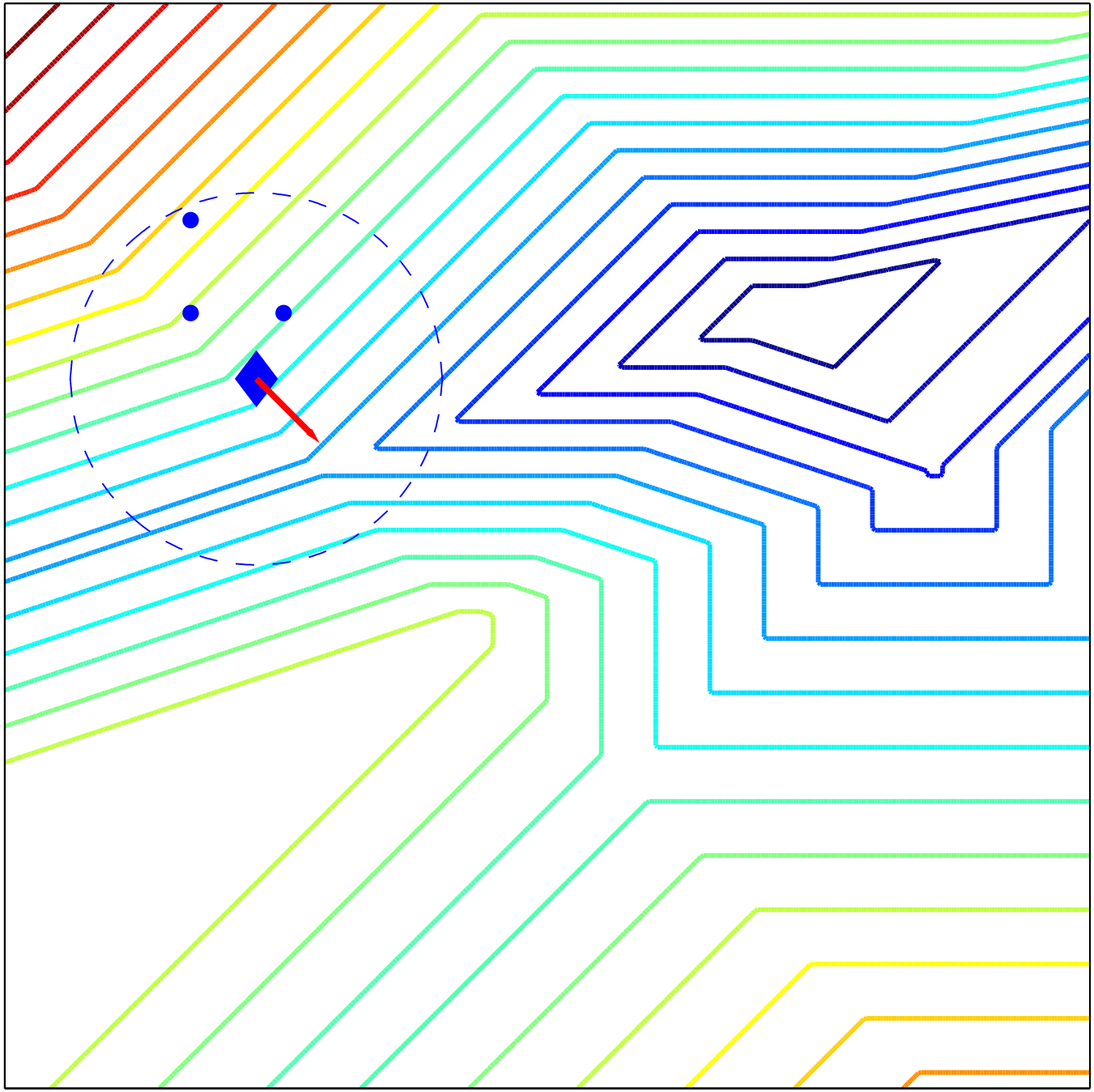


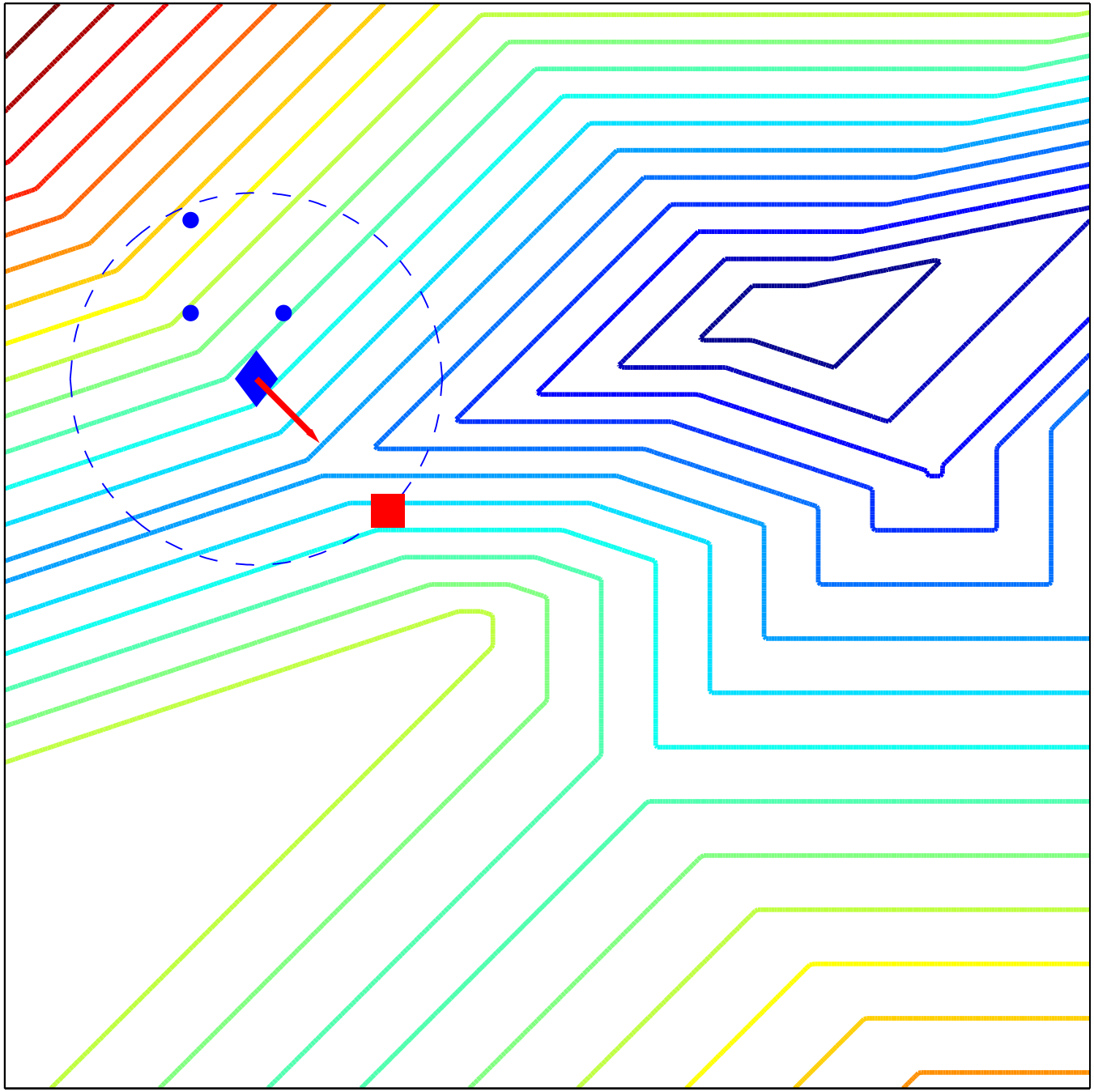


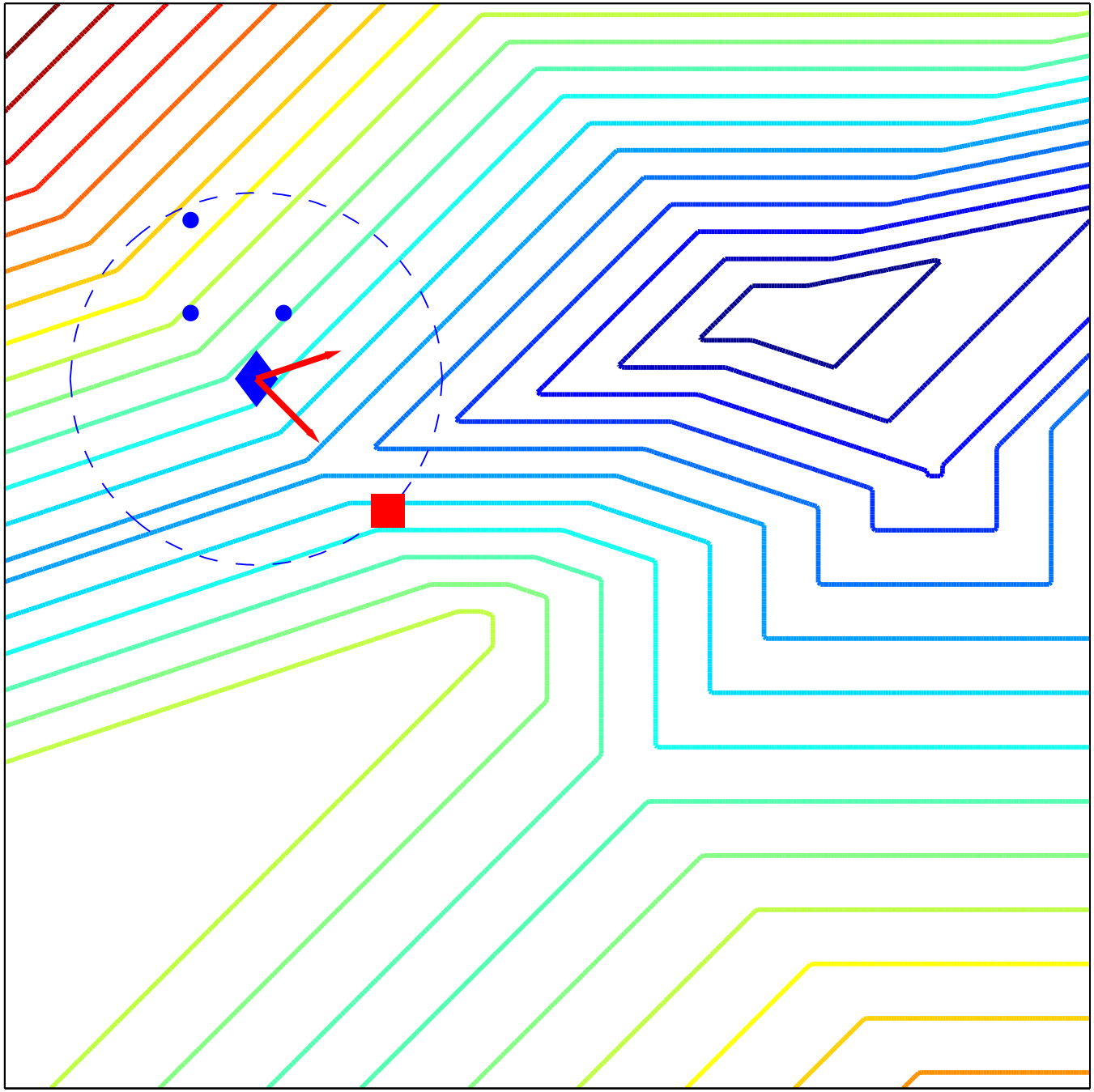


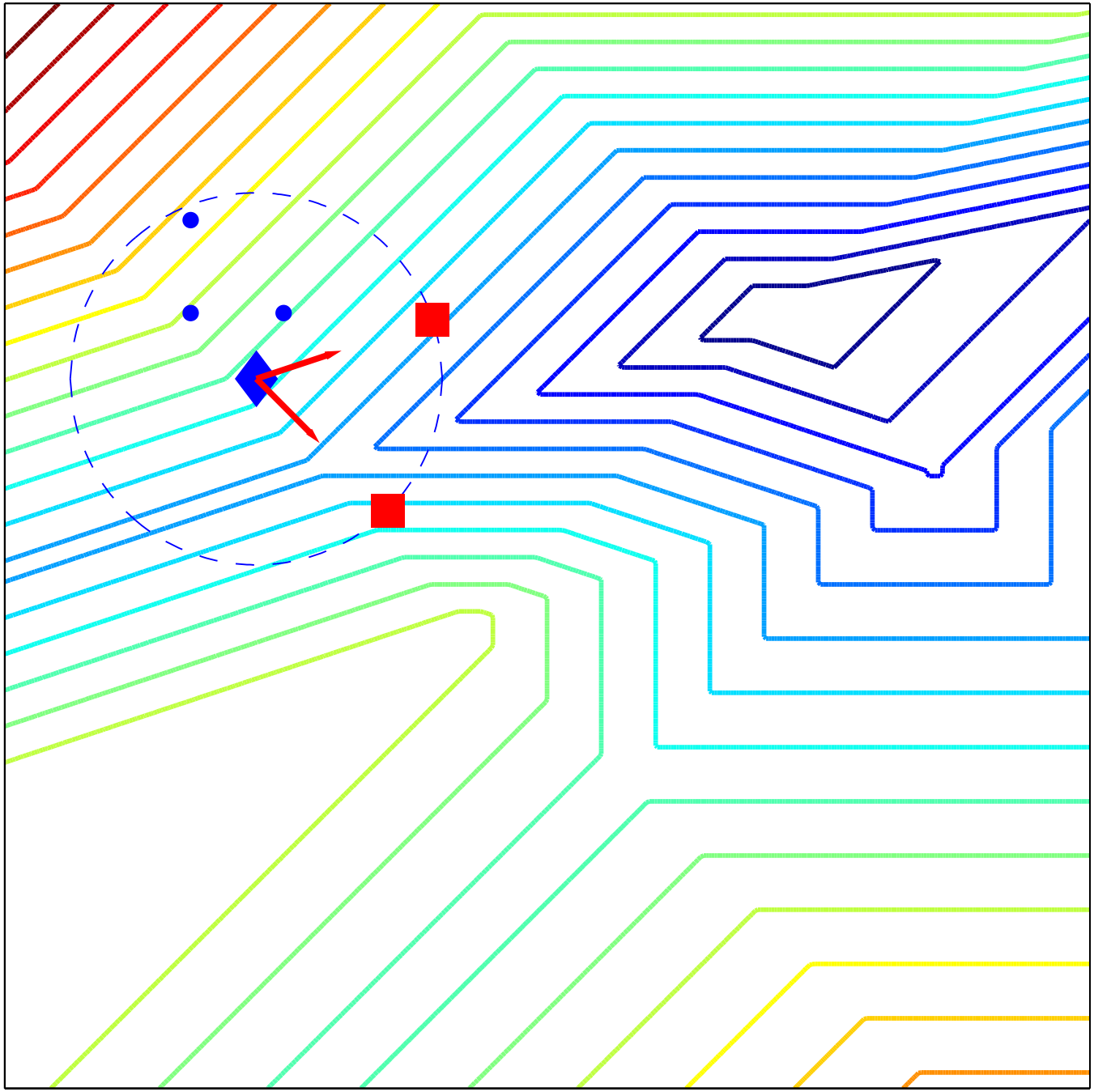


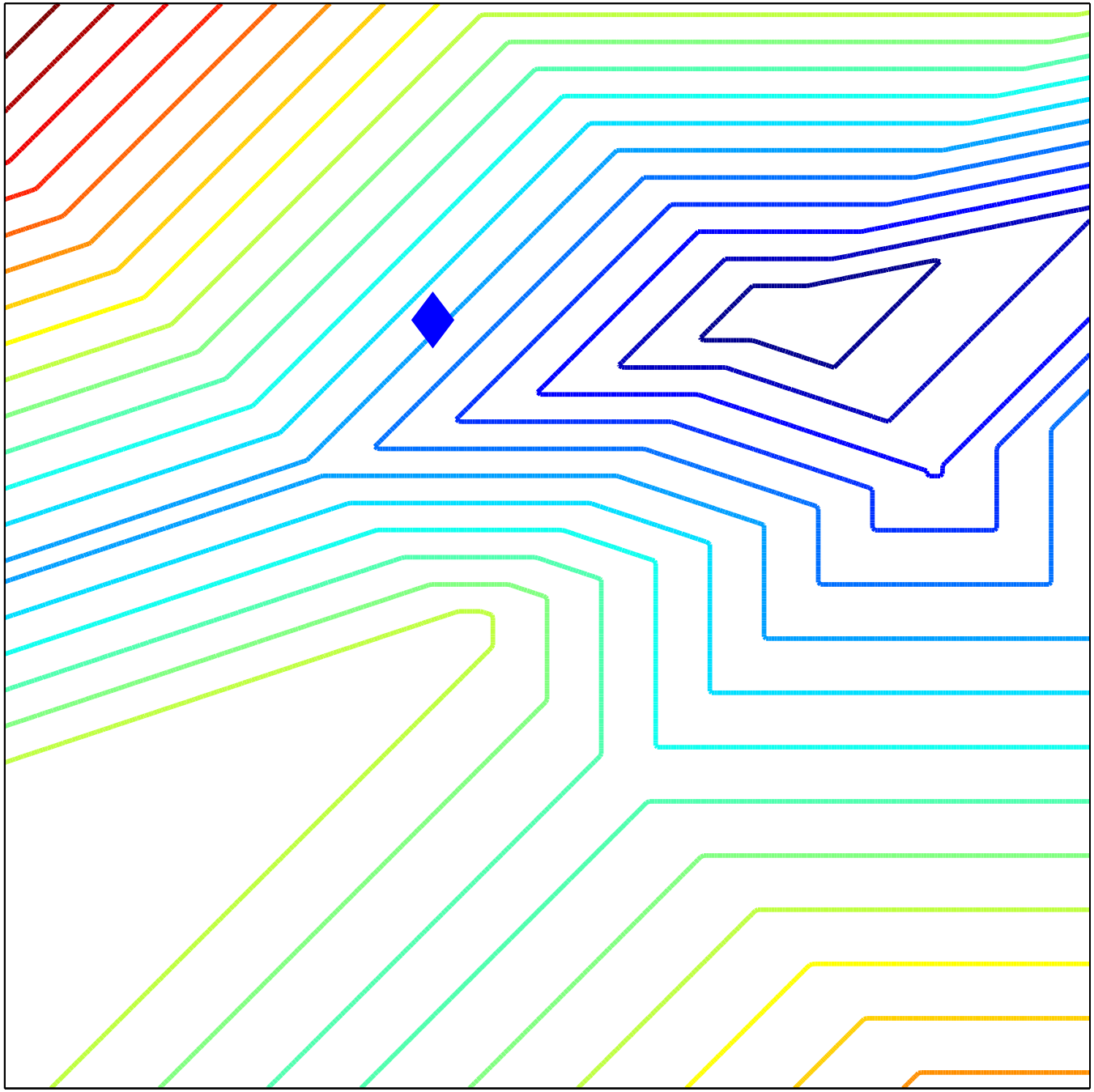


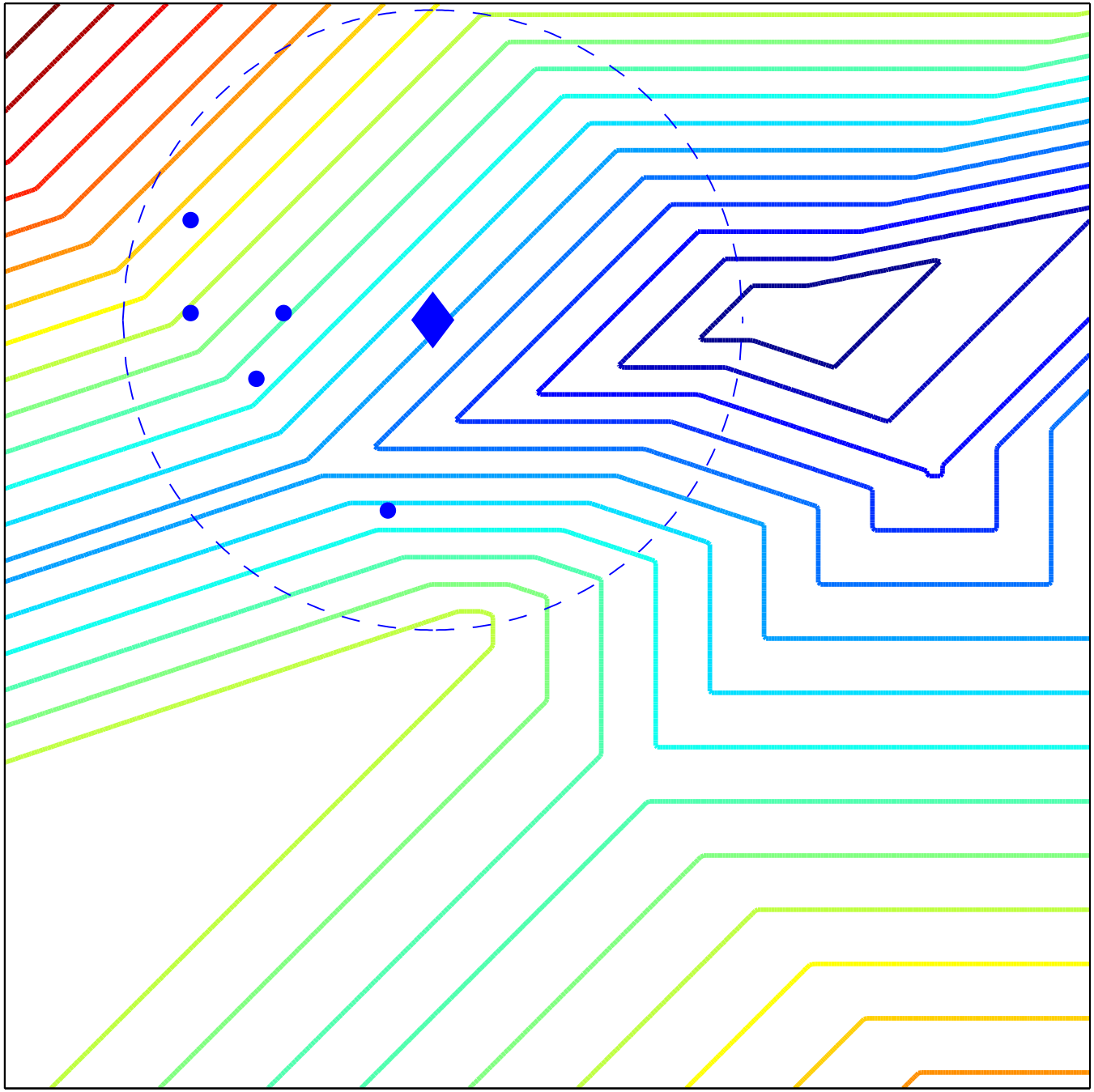


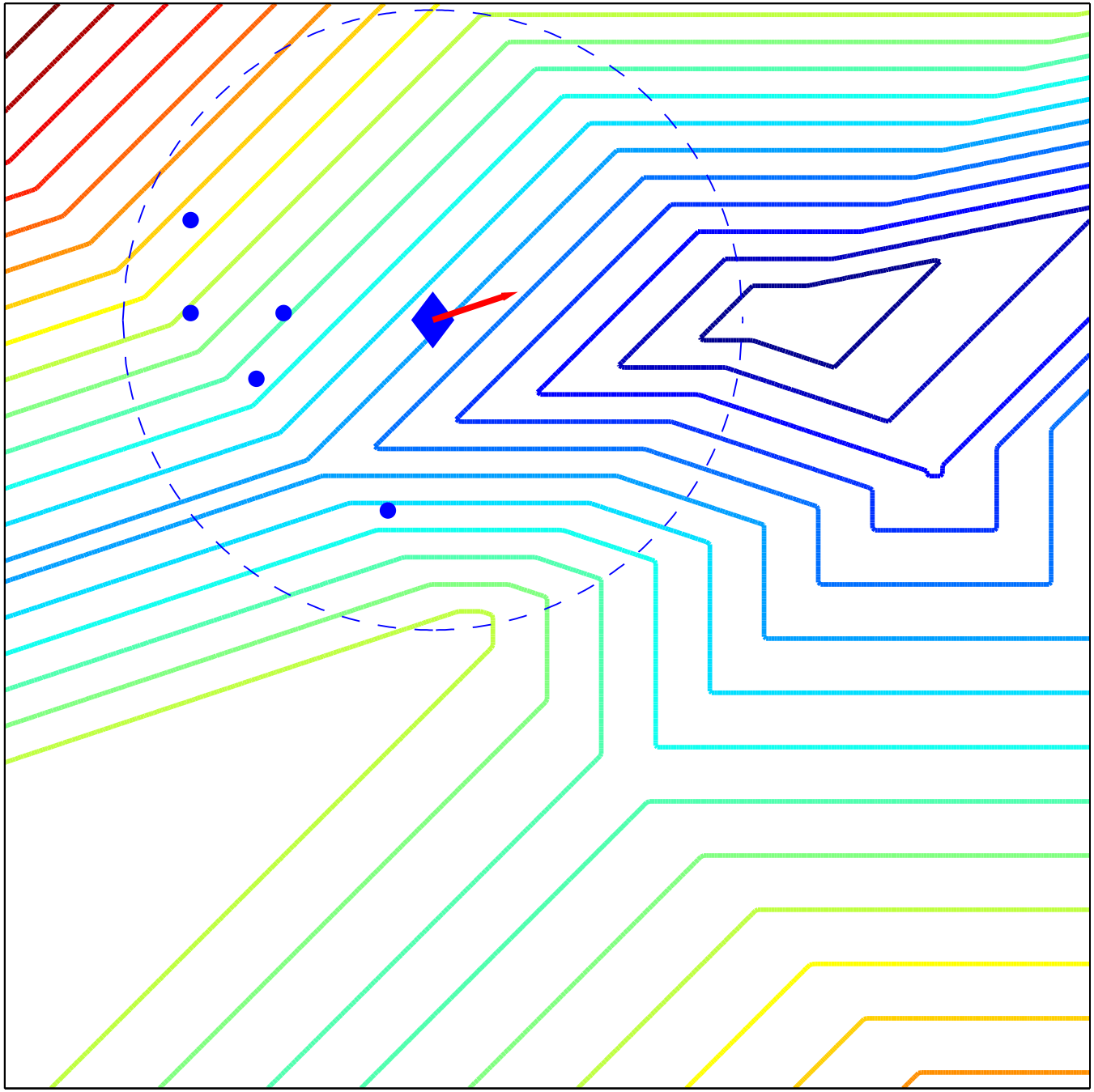


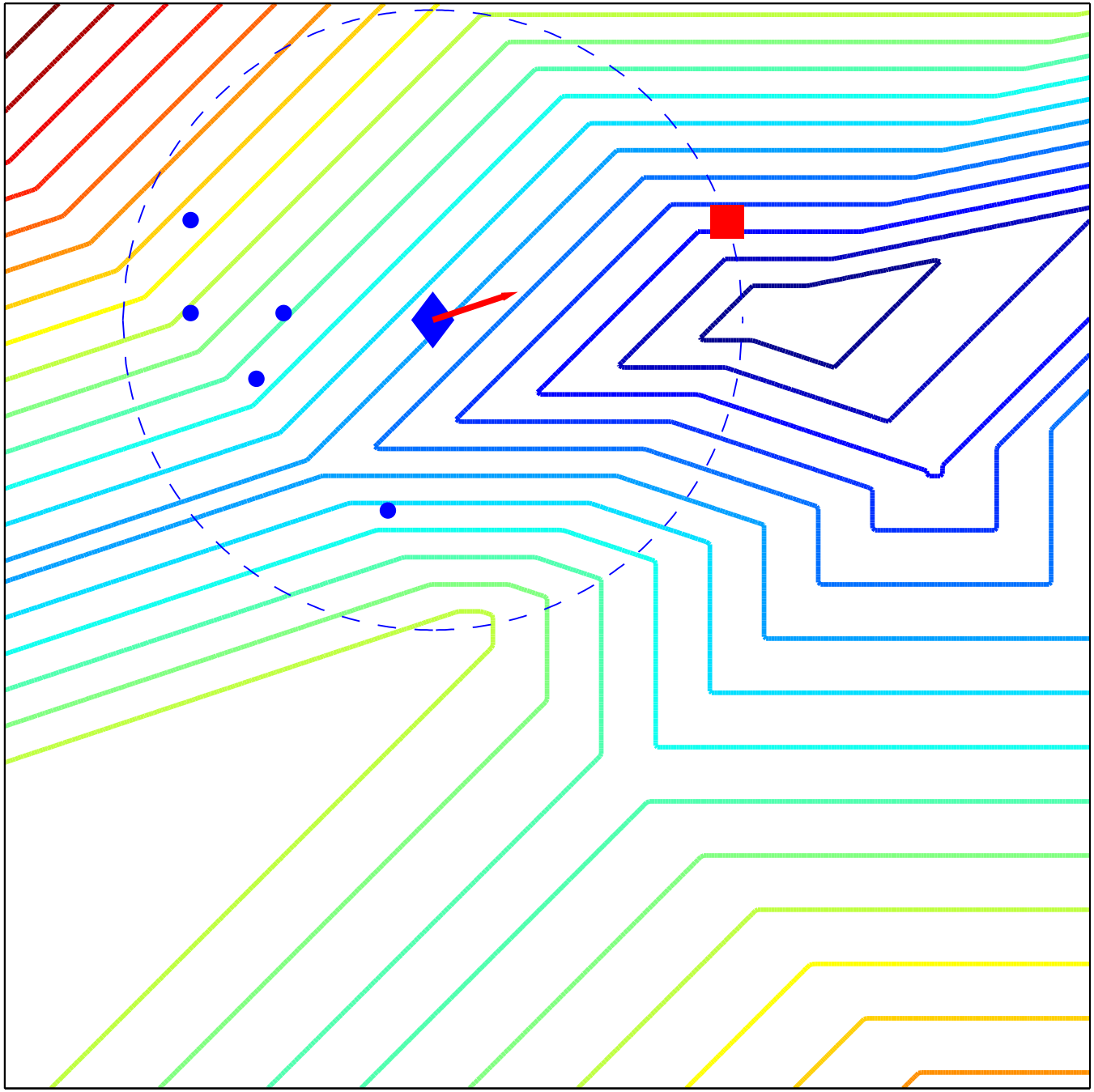


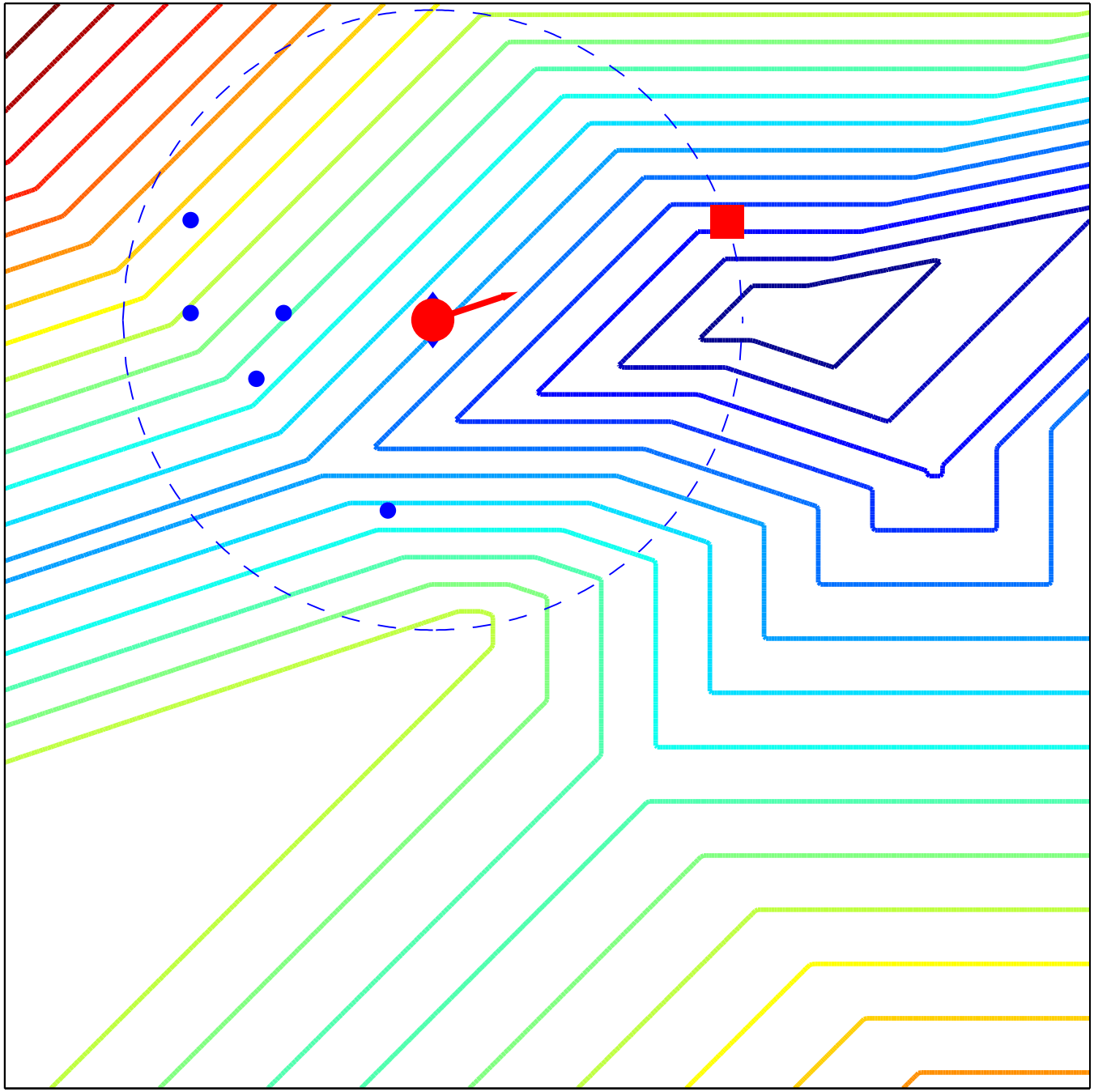


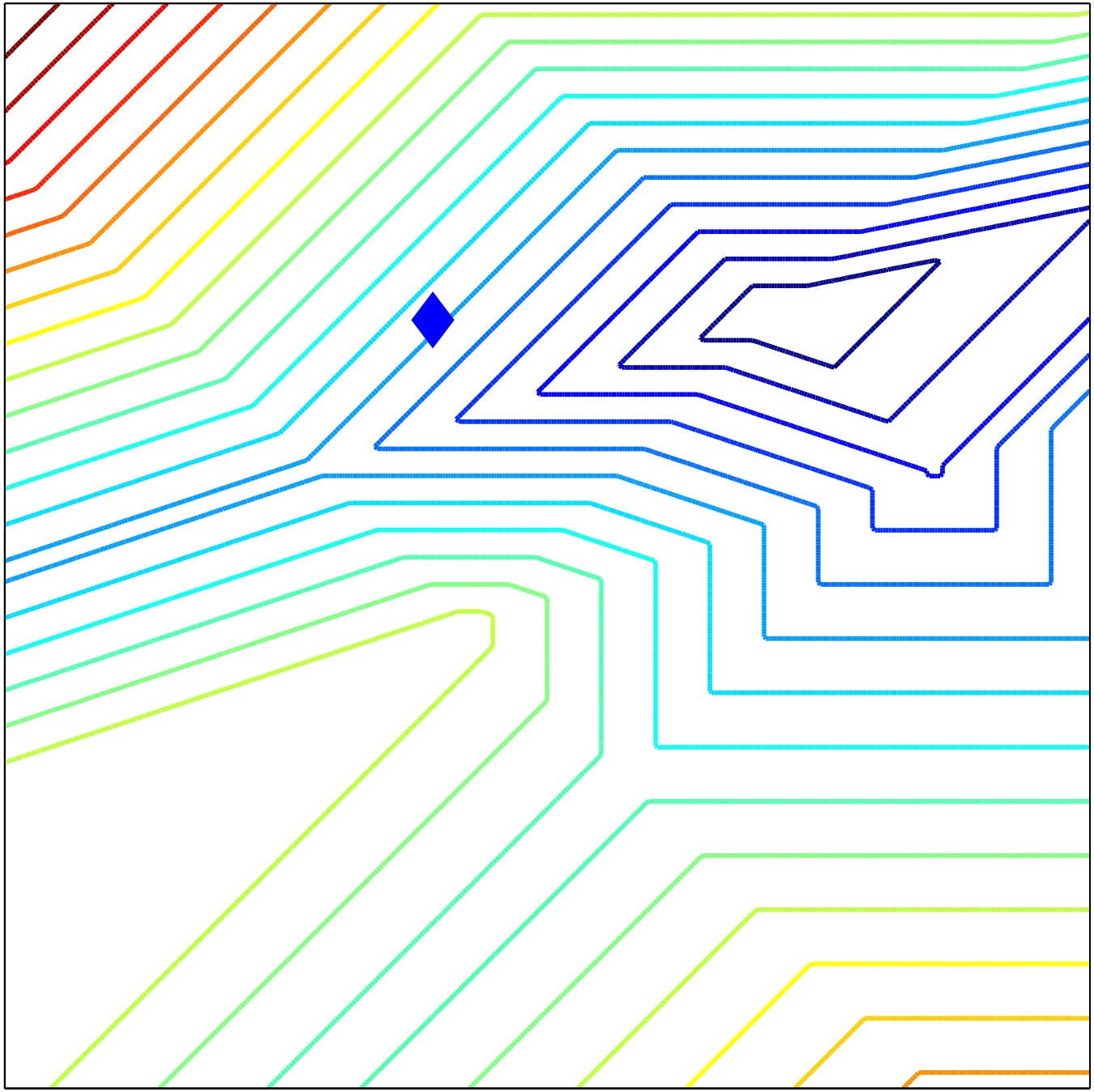


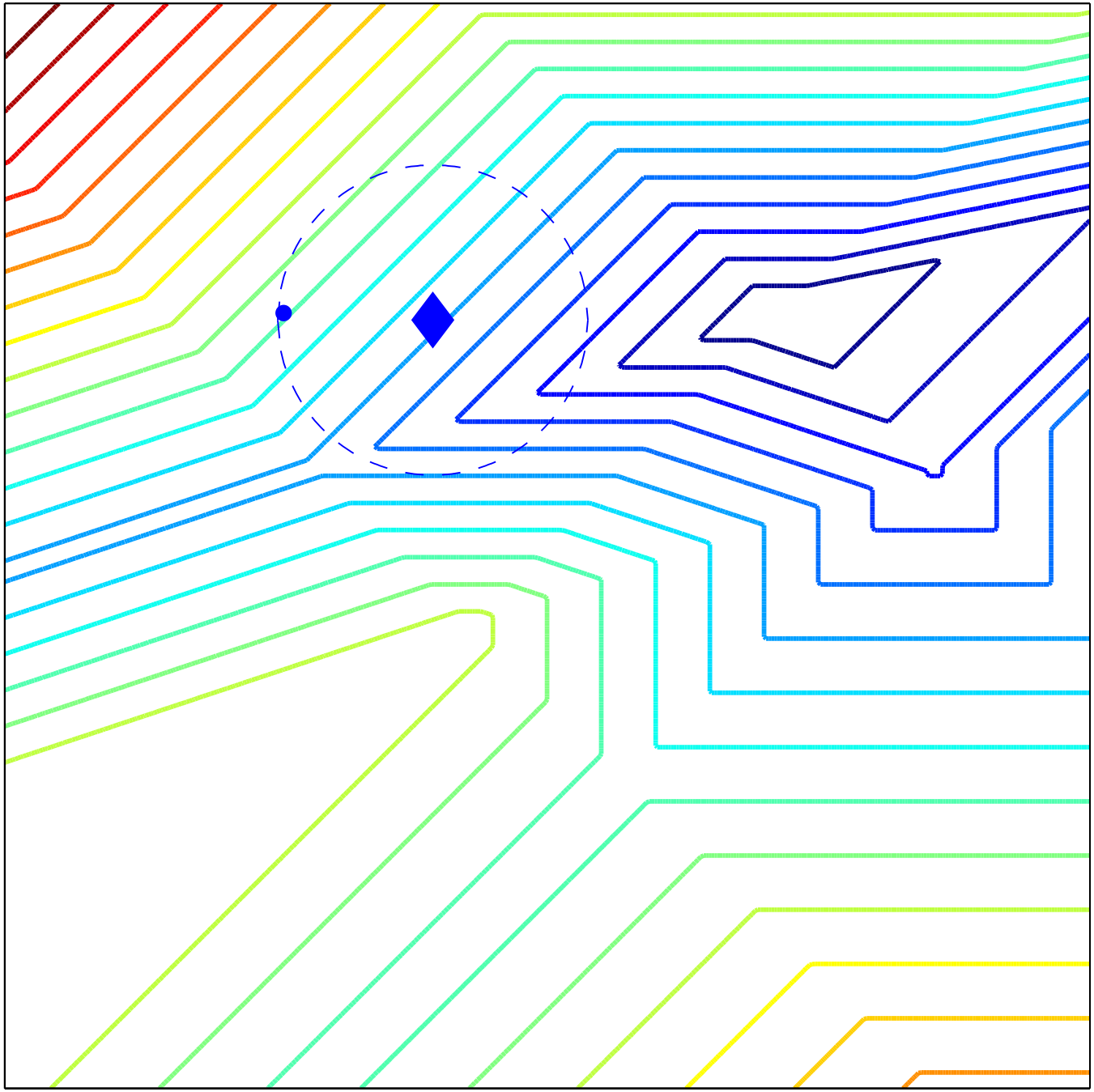


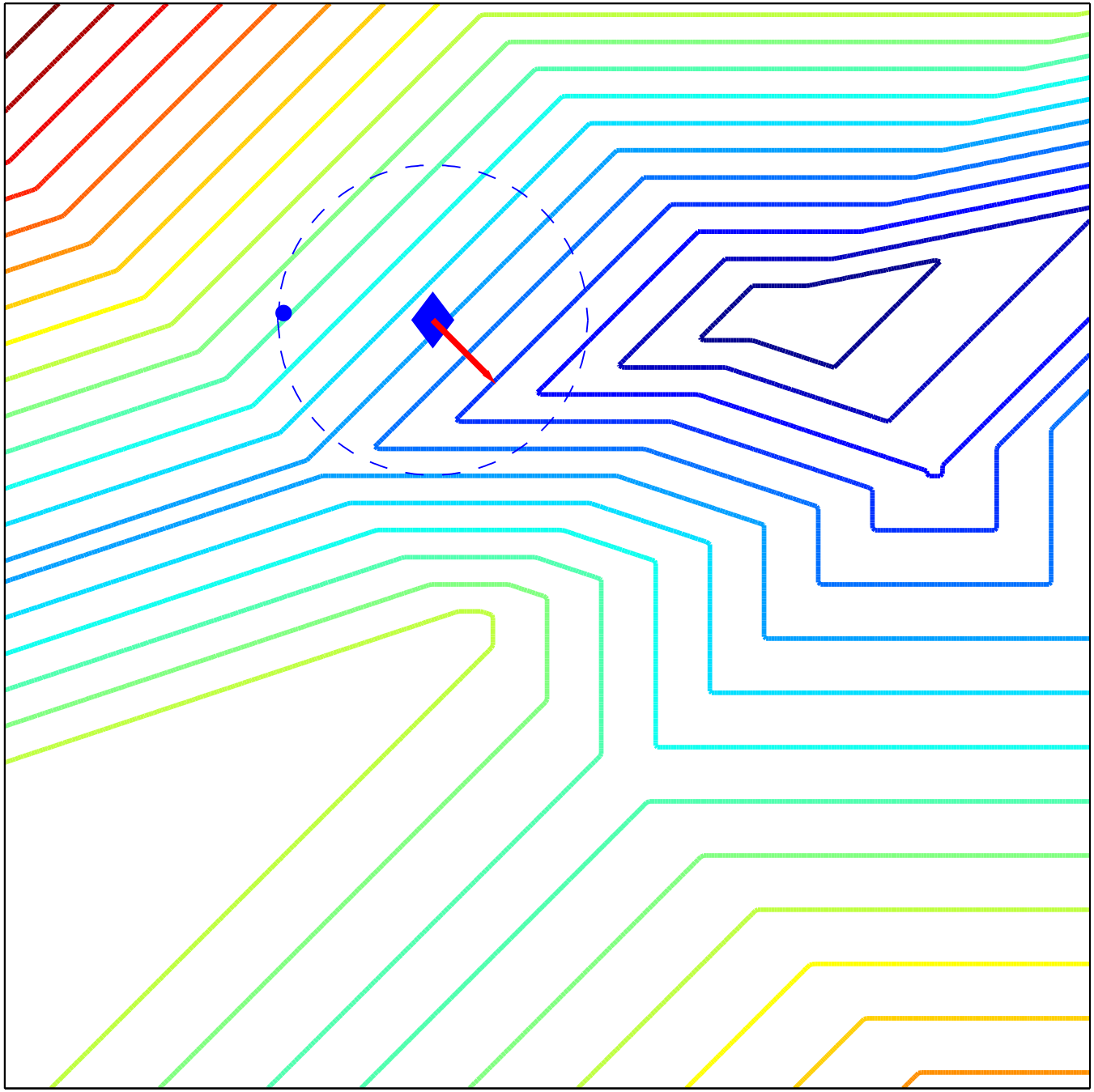


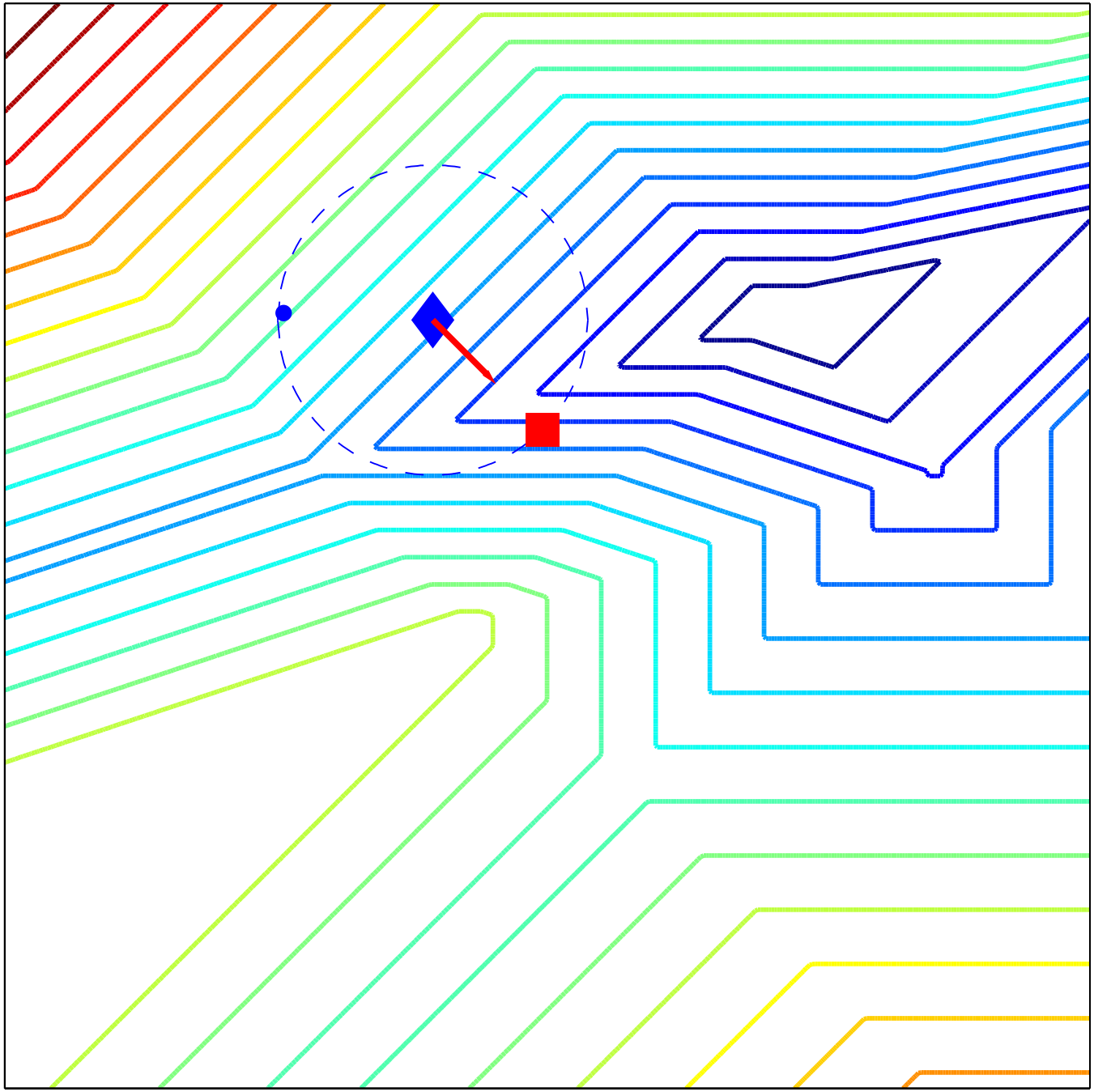


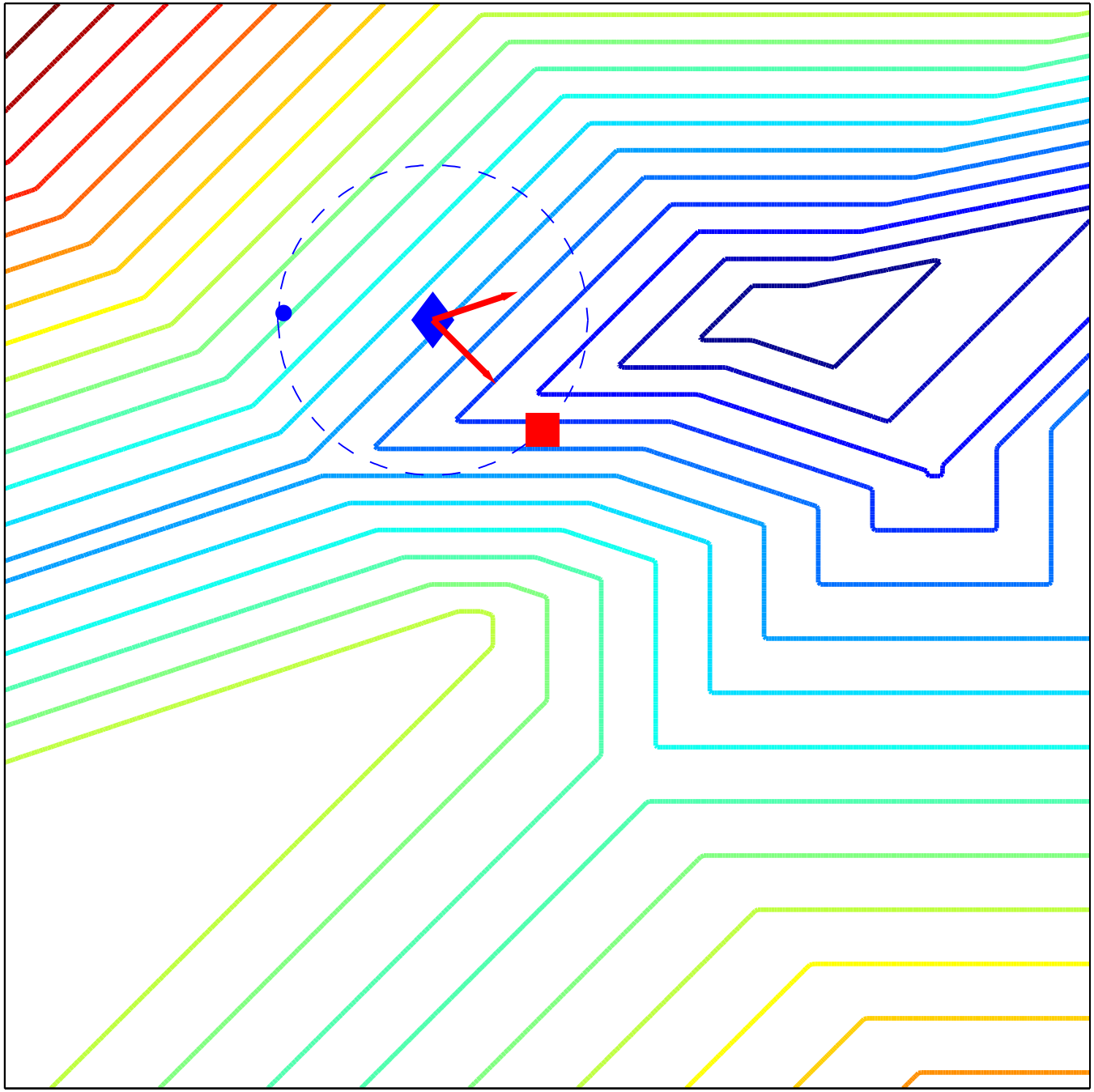


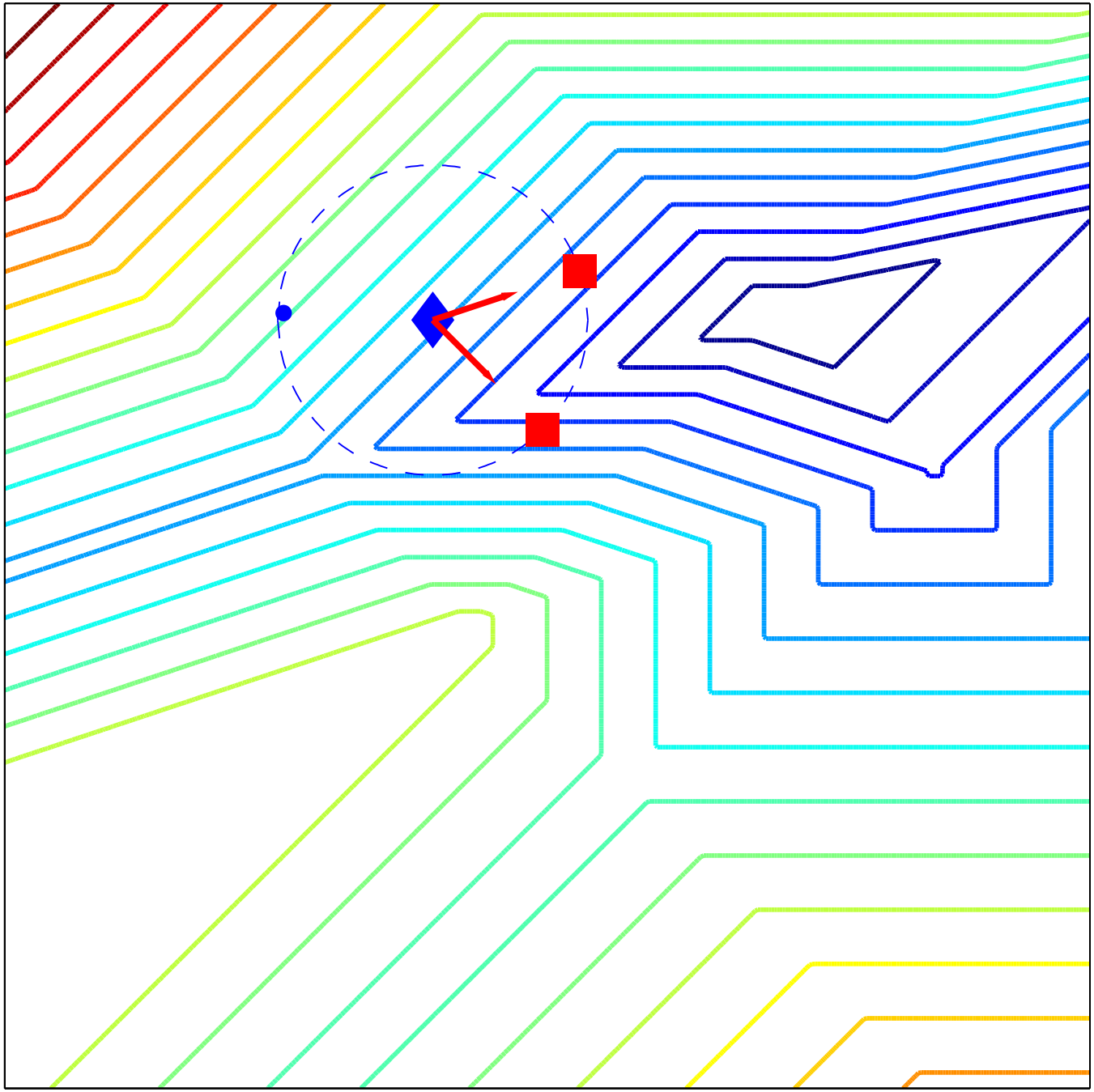


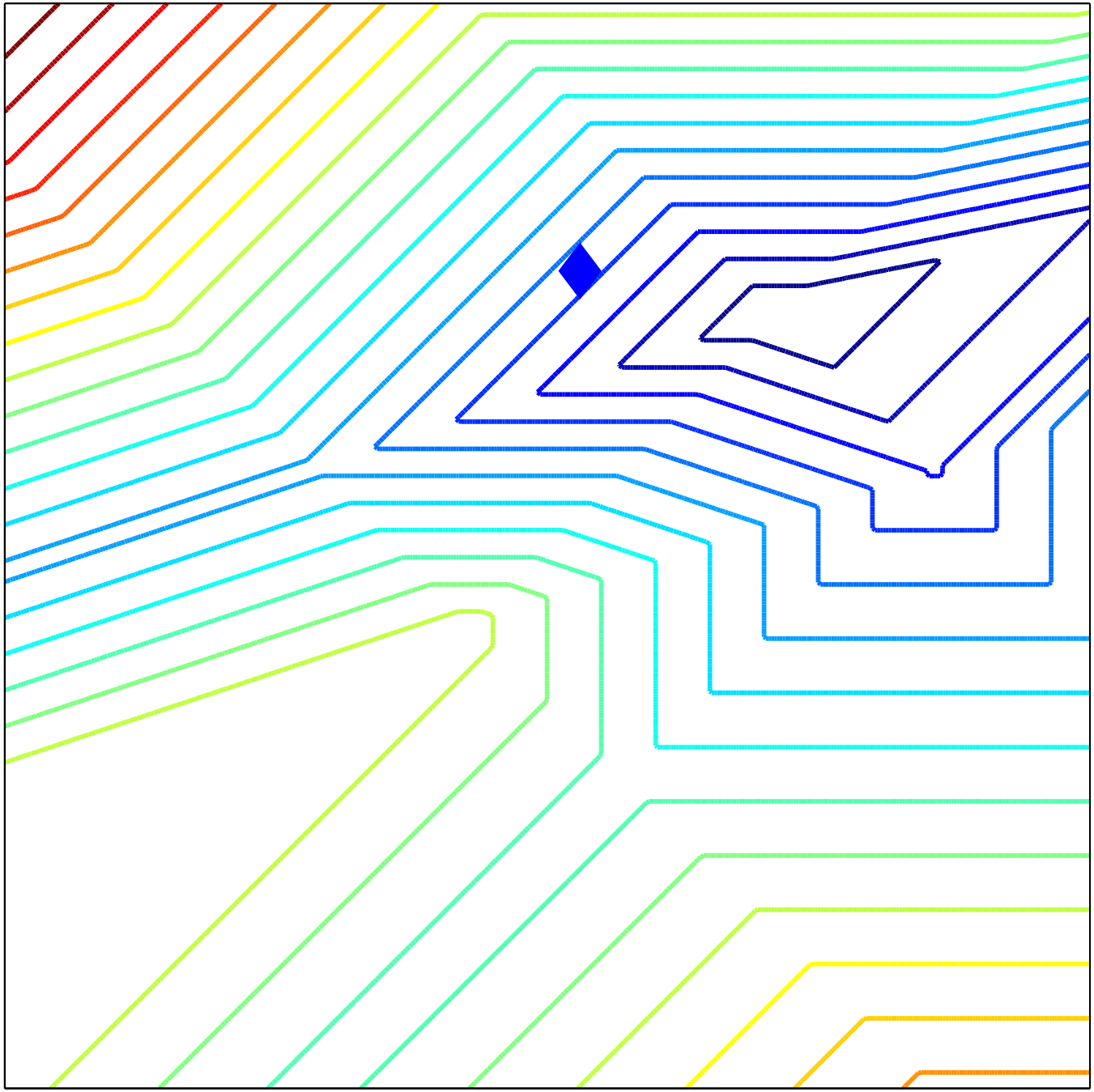


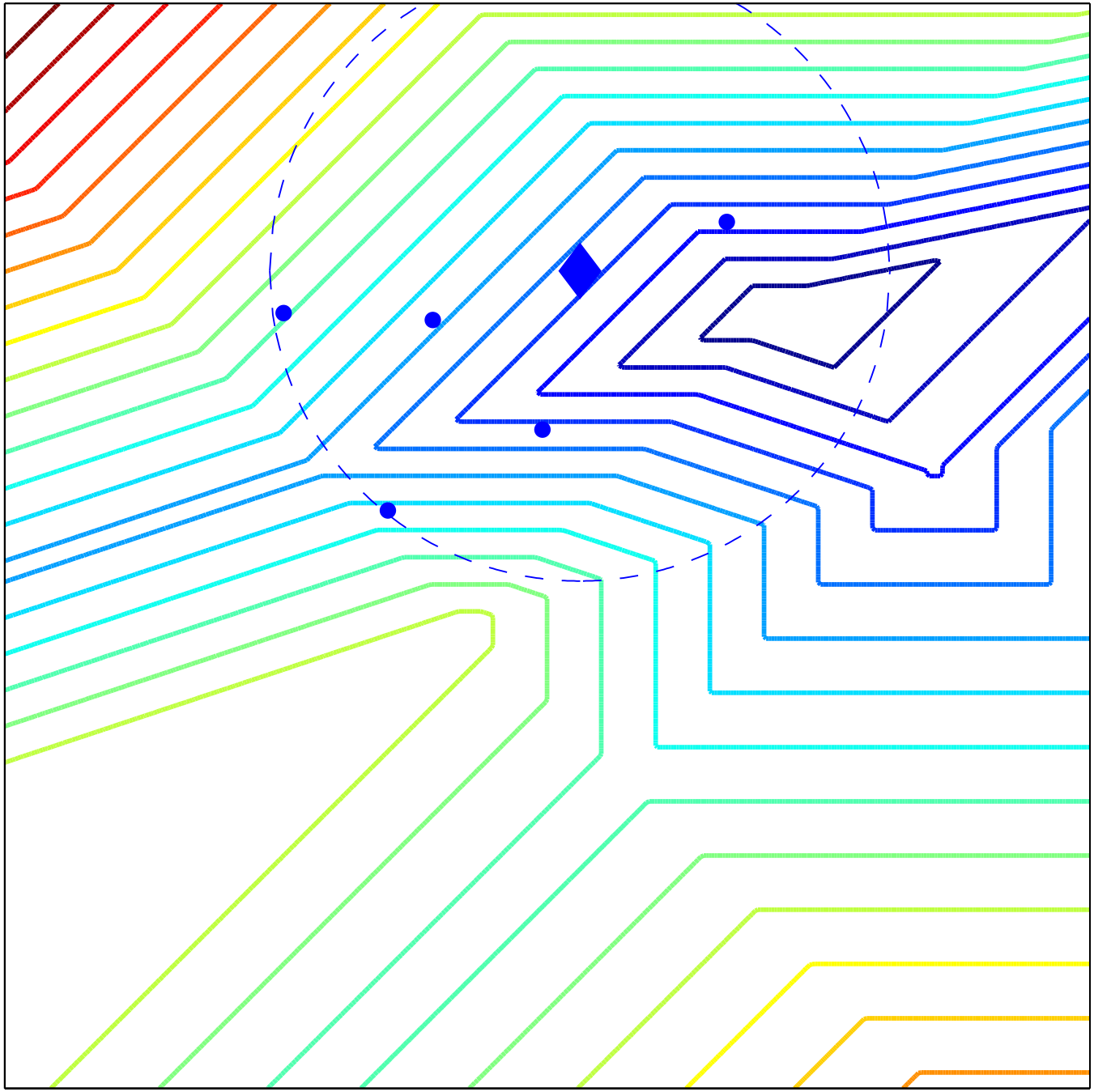


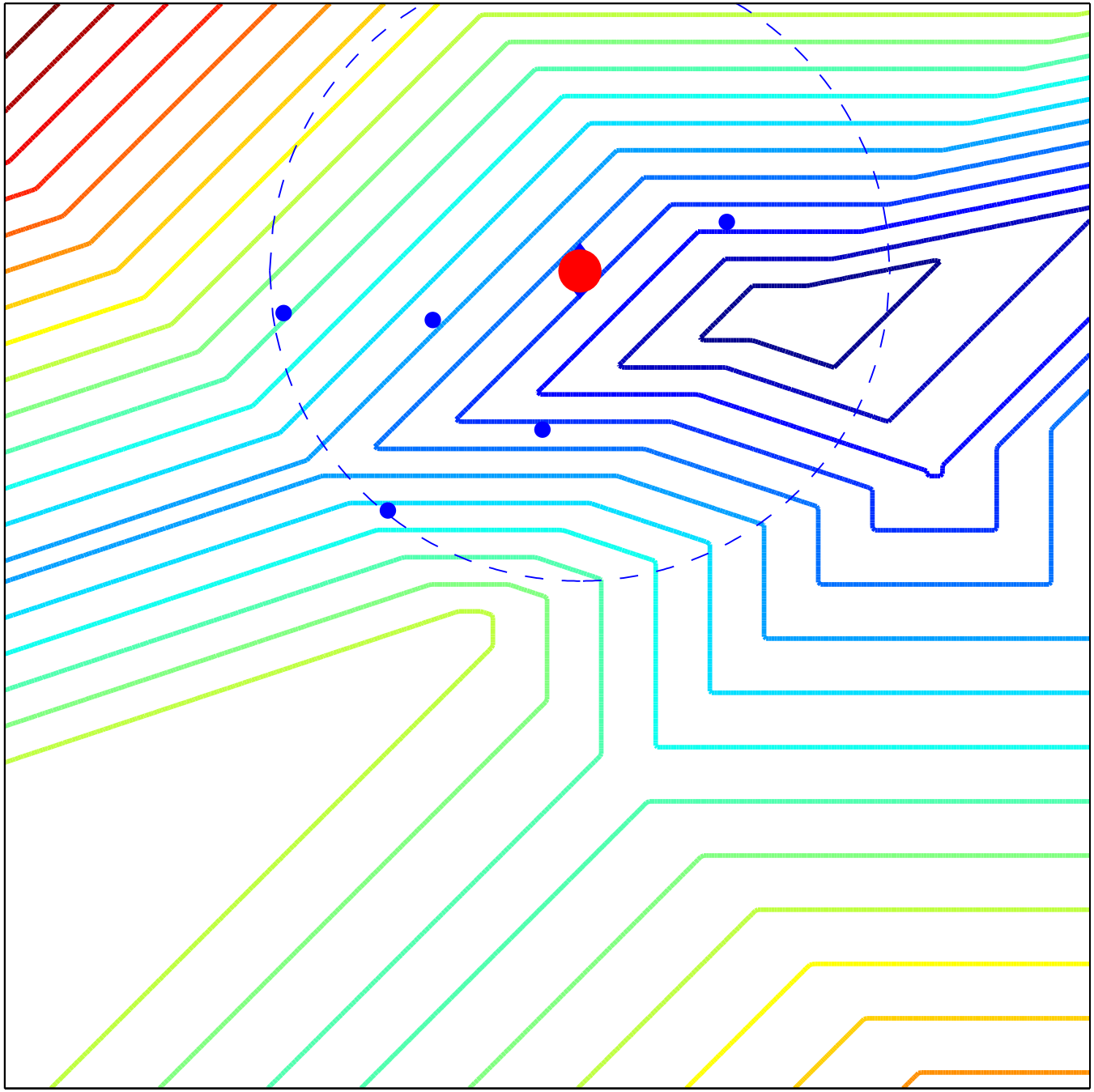


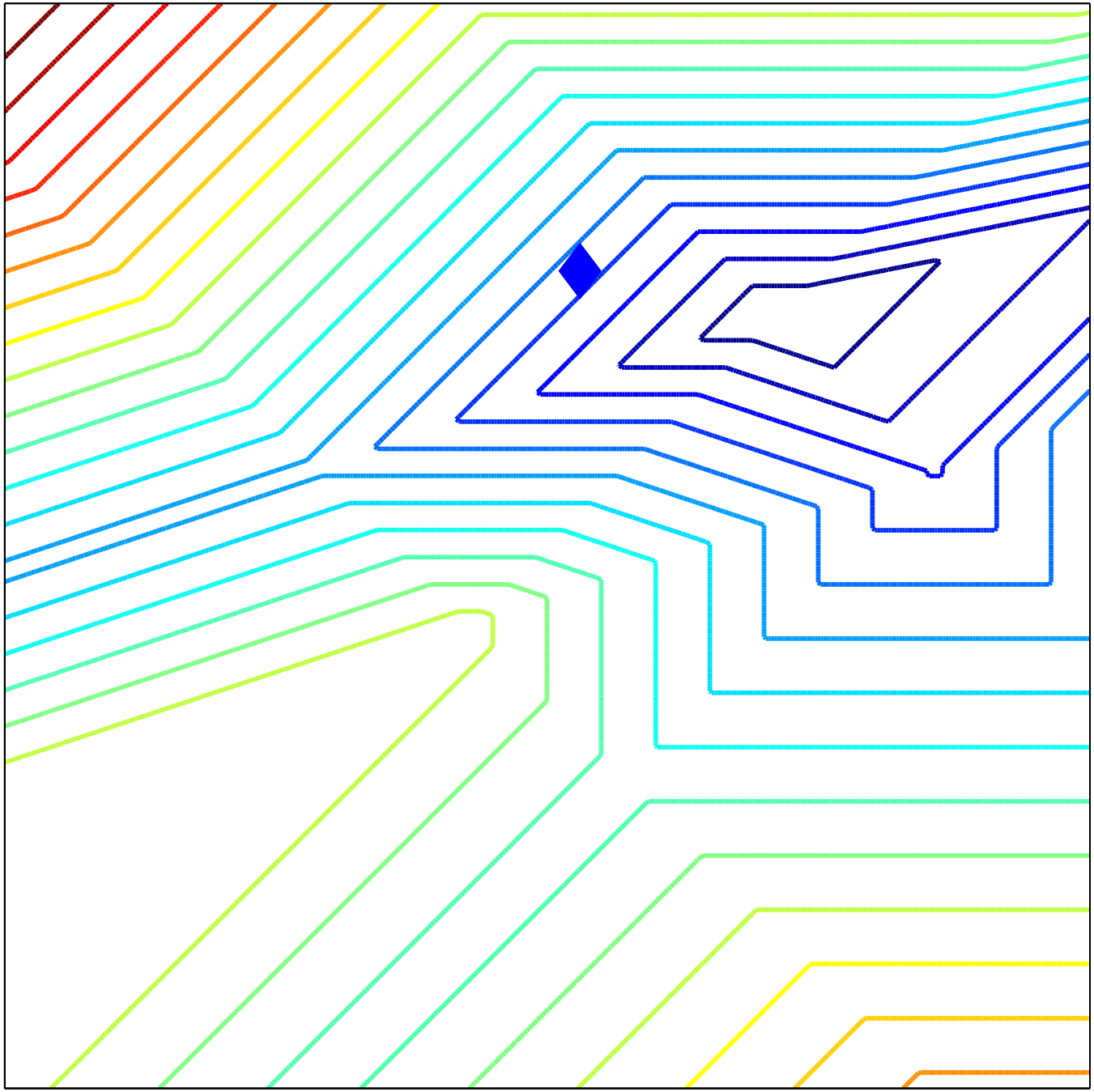


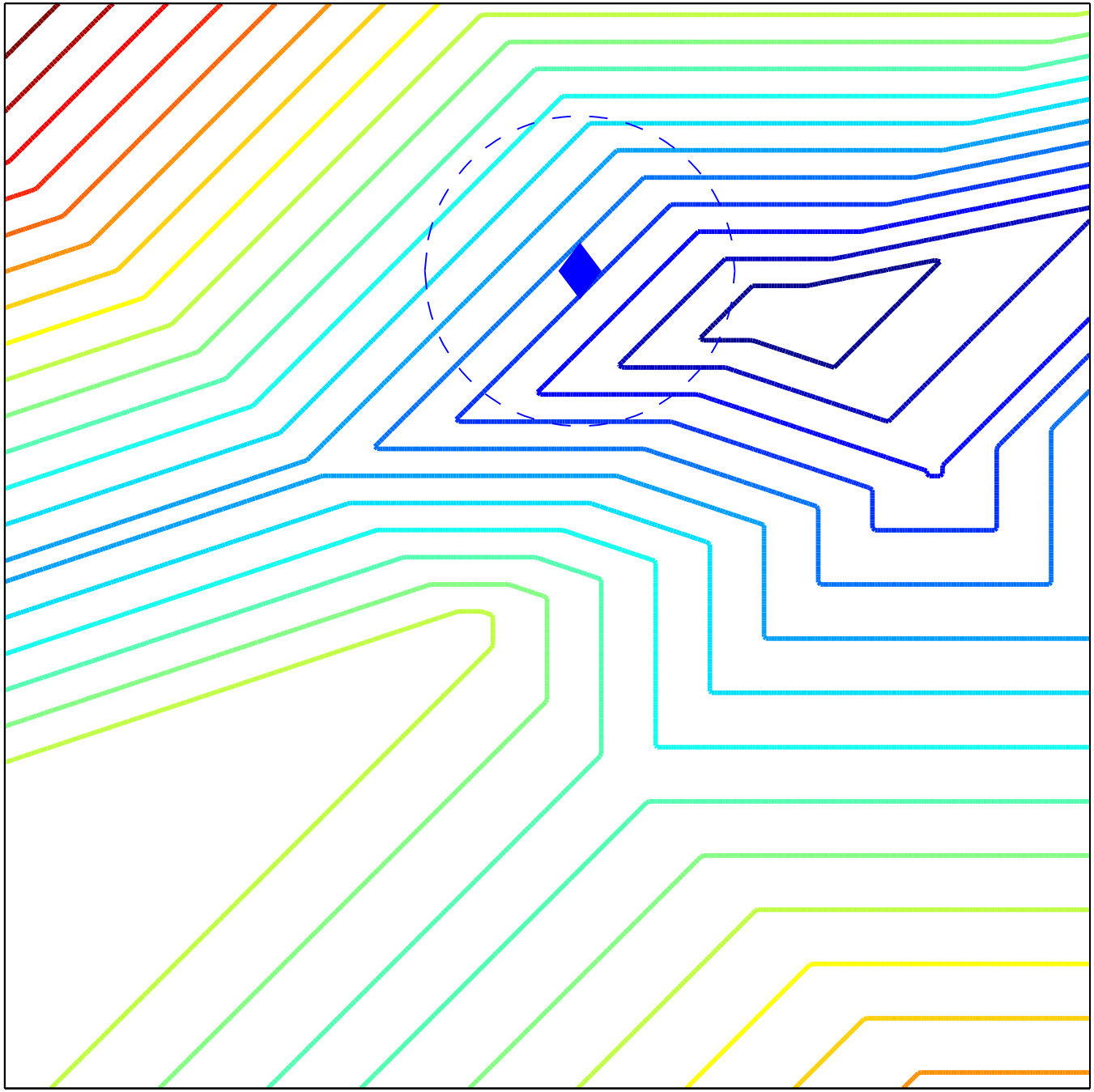


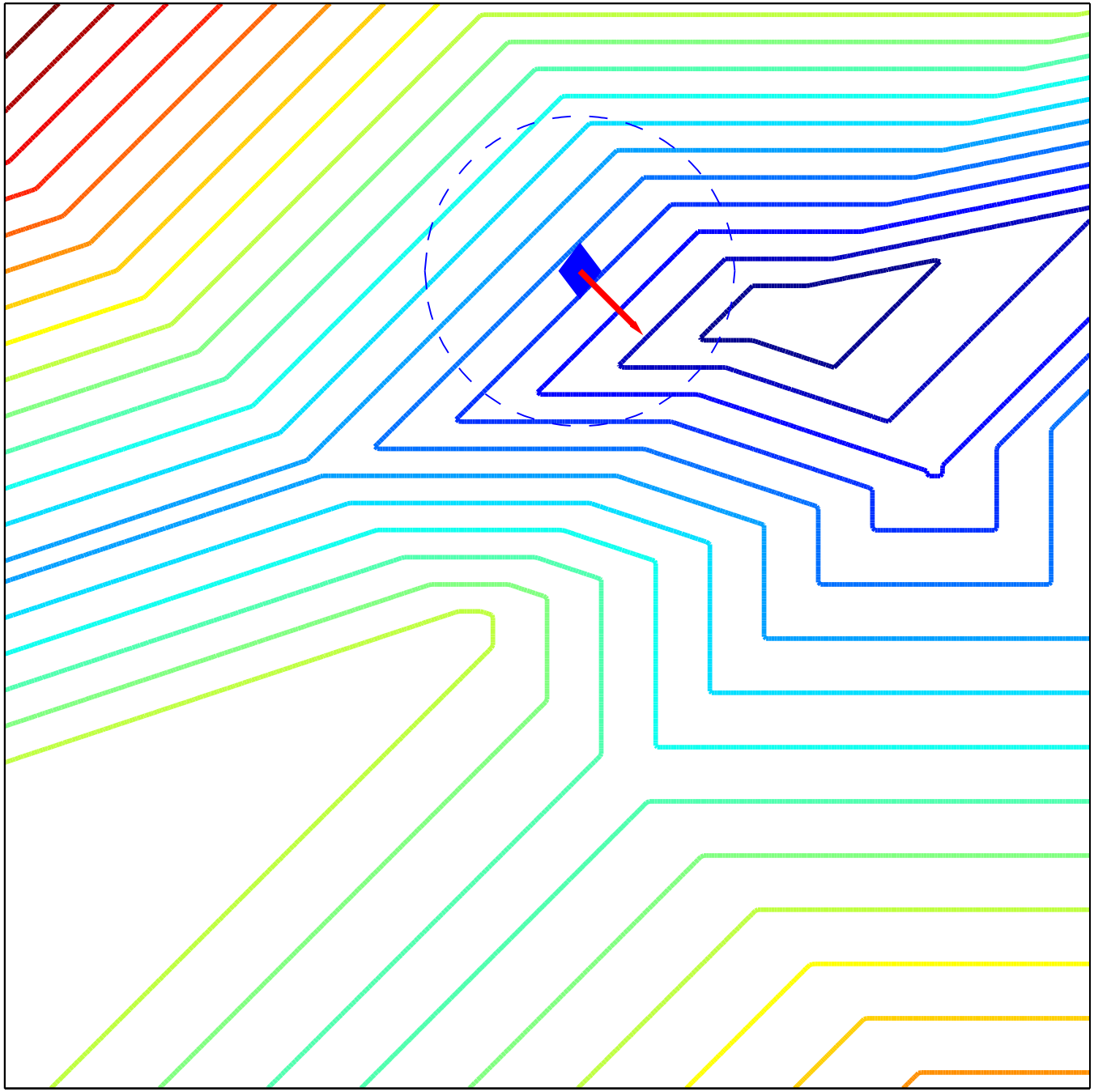


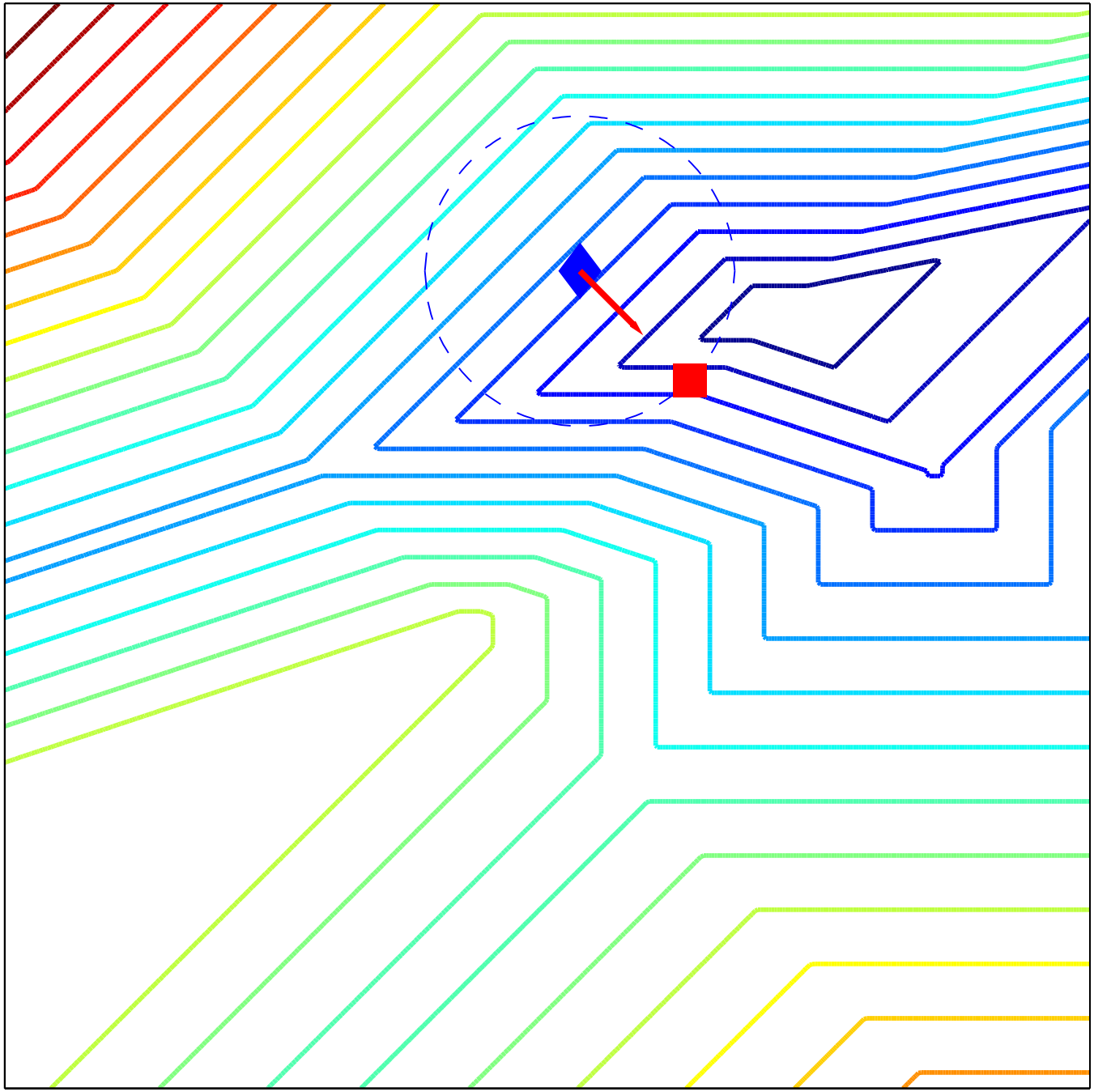


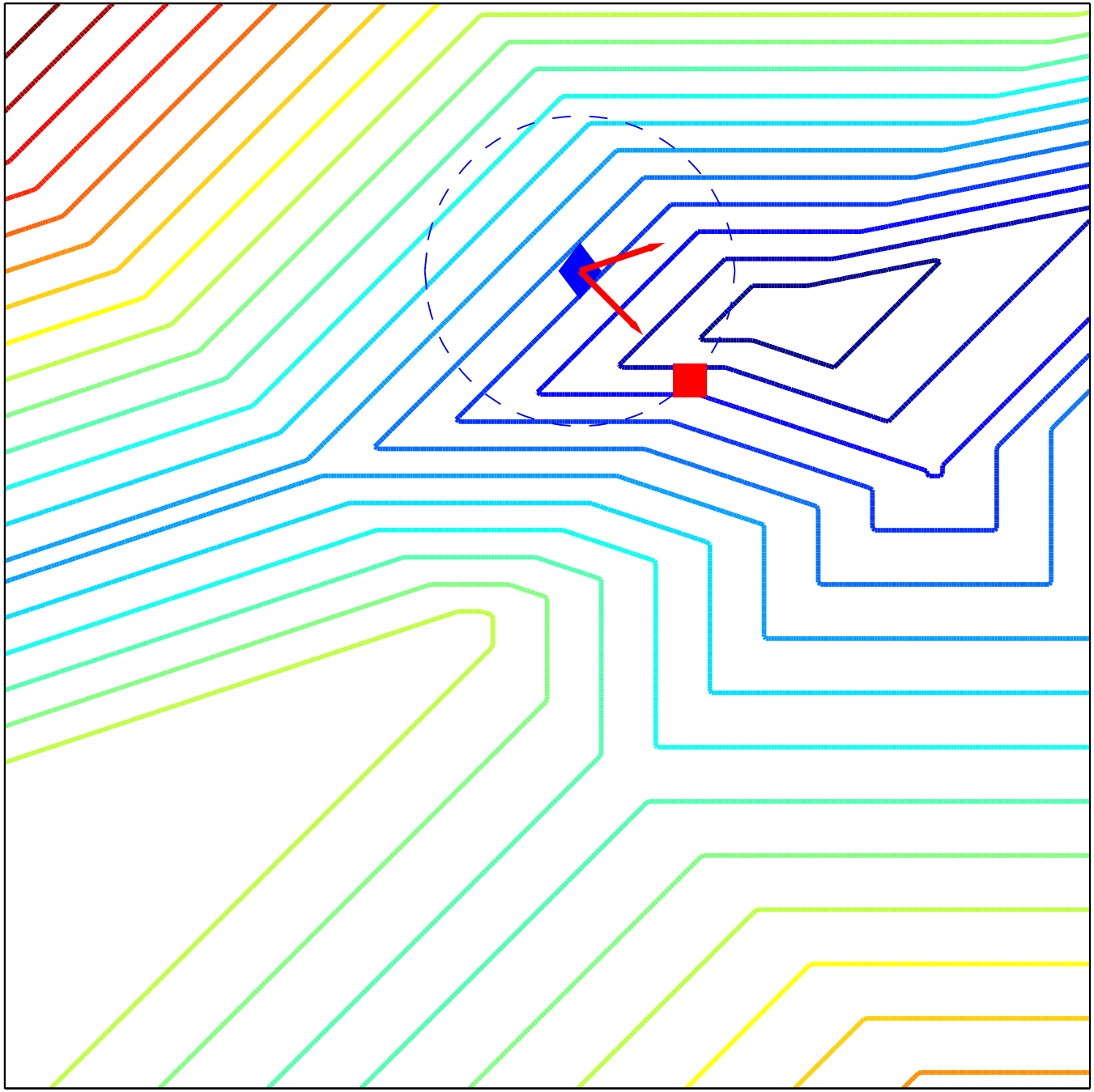


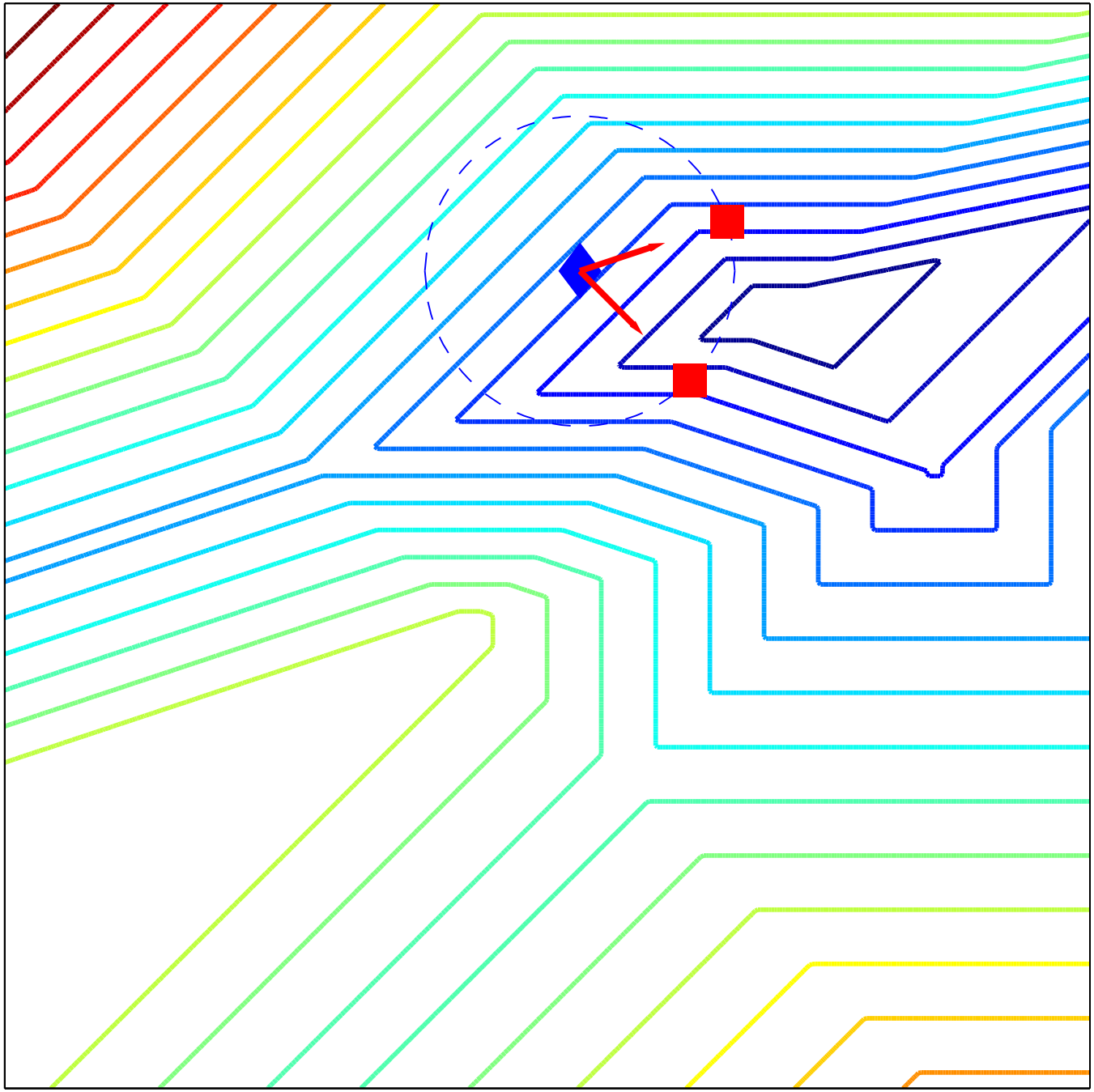


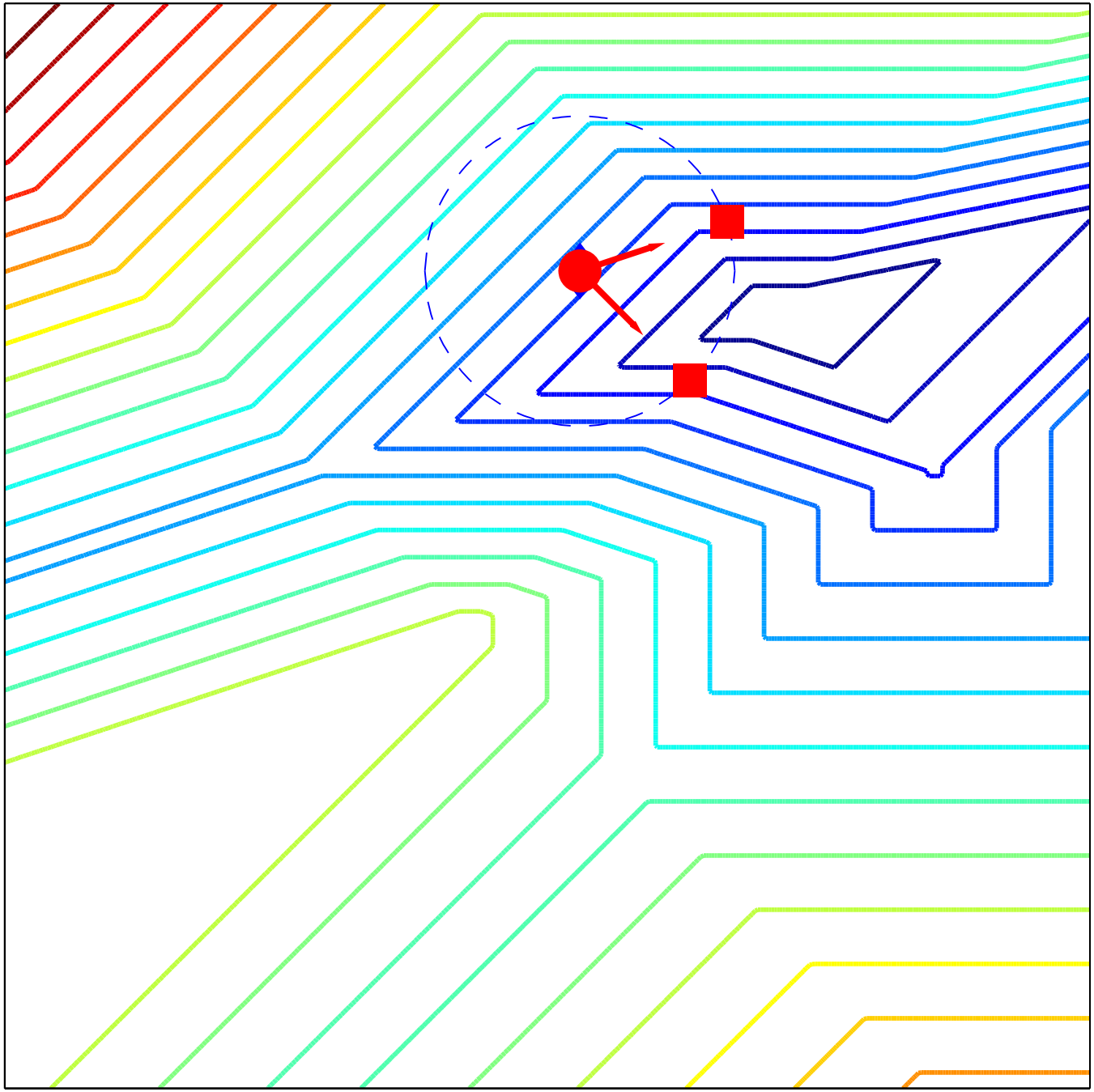












Convergence

- If the trust region radius Δ_k is a sufficiently small multiple of the master model gradient $\left\|g^k\right\|$, the iteration is guaranteed to be successful.
- $\lim_{k\to\infty} \Delta_k = 0$.
- Some subsequence of master model gradients g^k goes zero.
- Zero is in the generalized Clarke subdifferential of cluster points of any subsequence of iterates with master model gradients converging to zero.
- The same holds for cluster points of the sequence of MS4PL iterates.

Test problems

Let h be a censored ℓ_1-loss function. Given data $d \in \mathbb{R}^p$, censors $c \in \mathbb{R}^p$, and the mapping $F : \mathbb{R}^n \to \mathbb{R}^p$, we define

$$f(x) = \sum_{i=1}^{p} |d_i - \max\{F_i(x), c_i\}| .$$

That is, $\psi = 0$, and

$$h(y) = \sum_{i=1}^{p} |d_i - \max\{y_i, c_i\}| .$$

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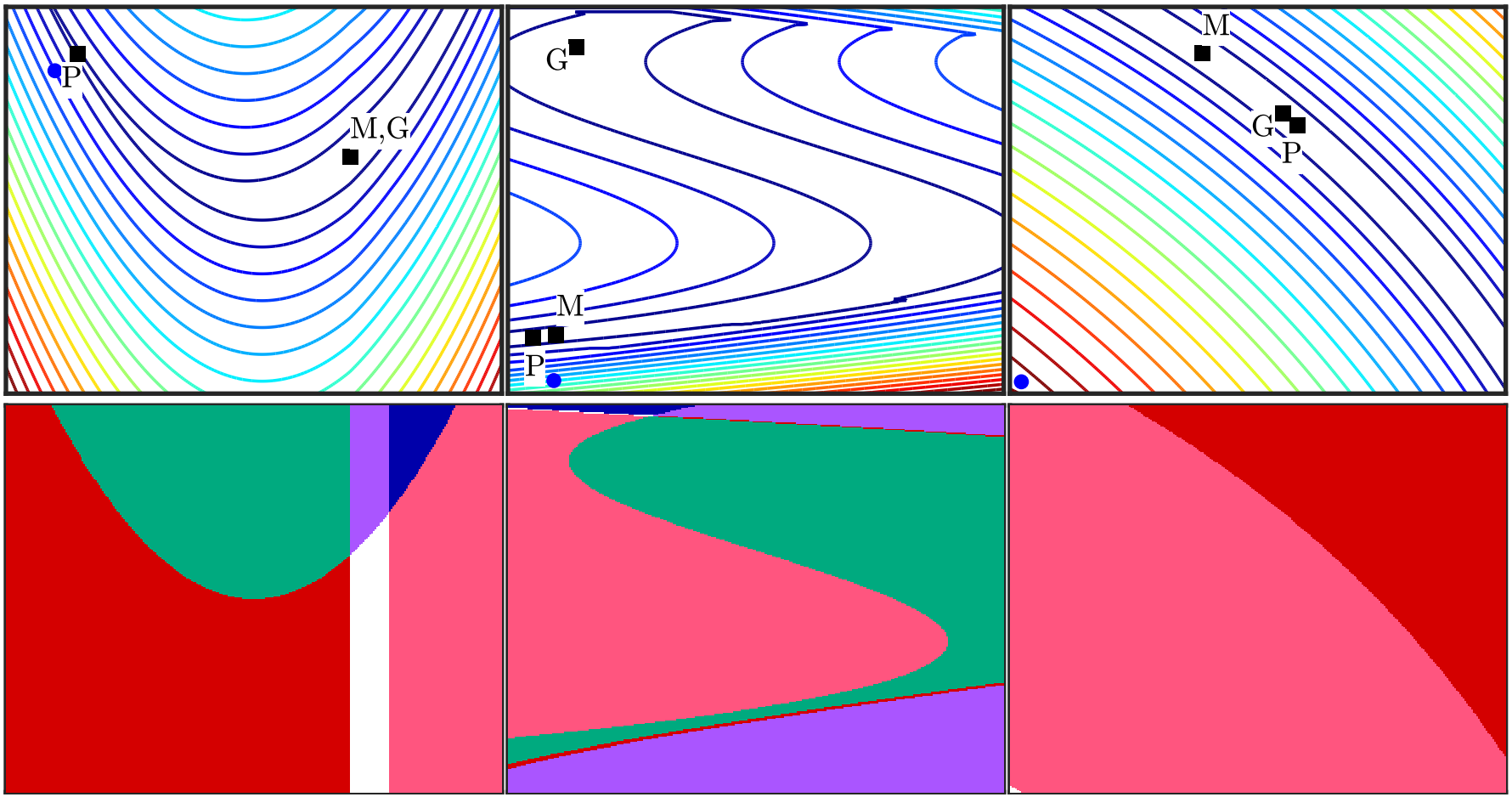
Define F to be the 53 vector mapping in the Móre and Wild benchmarking set. $2 \leq n \leq 12$, $2 \leq p \leq 45$.

Examples



Algorithms to compare

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MS4PL-2 Using manifolds in $\mathcal{B}(x^k, \Delta_k)$ [F, knowledge of h]

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MS4PL-1-grad Using manifolds at x^k [F, knowledge of h, ∇F for models]

Tests

f test A method s solves a problem p to a level τ after j function evaluations if

$$f(x^0) - f(x^j) \geq (1 - \tau)(f(x^0) - \tilde{f}_p)$$

x^0 is the problem's starting point, and $\tilde{f}_p$ is the best-found function value.

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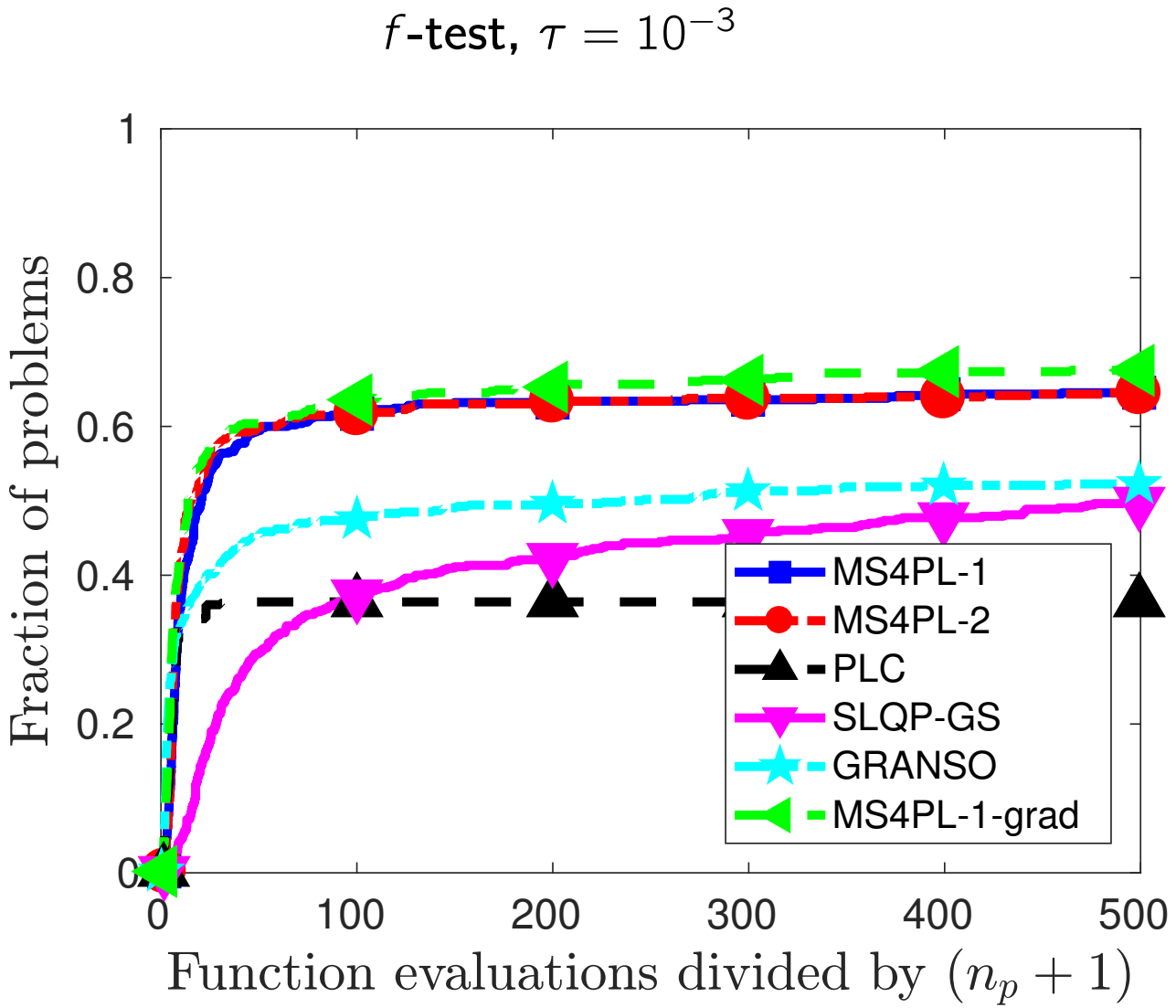
Data profiles



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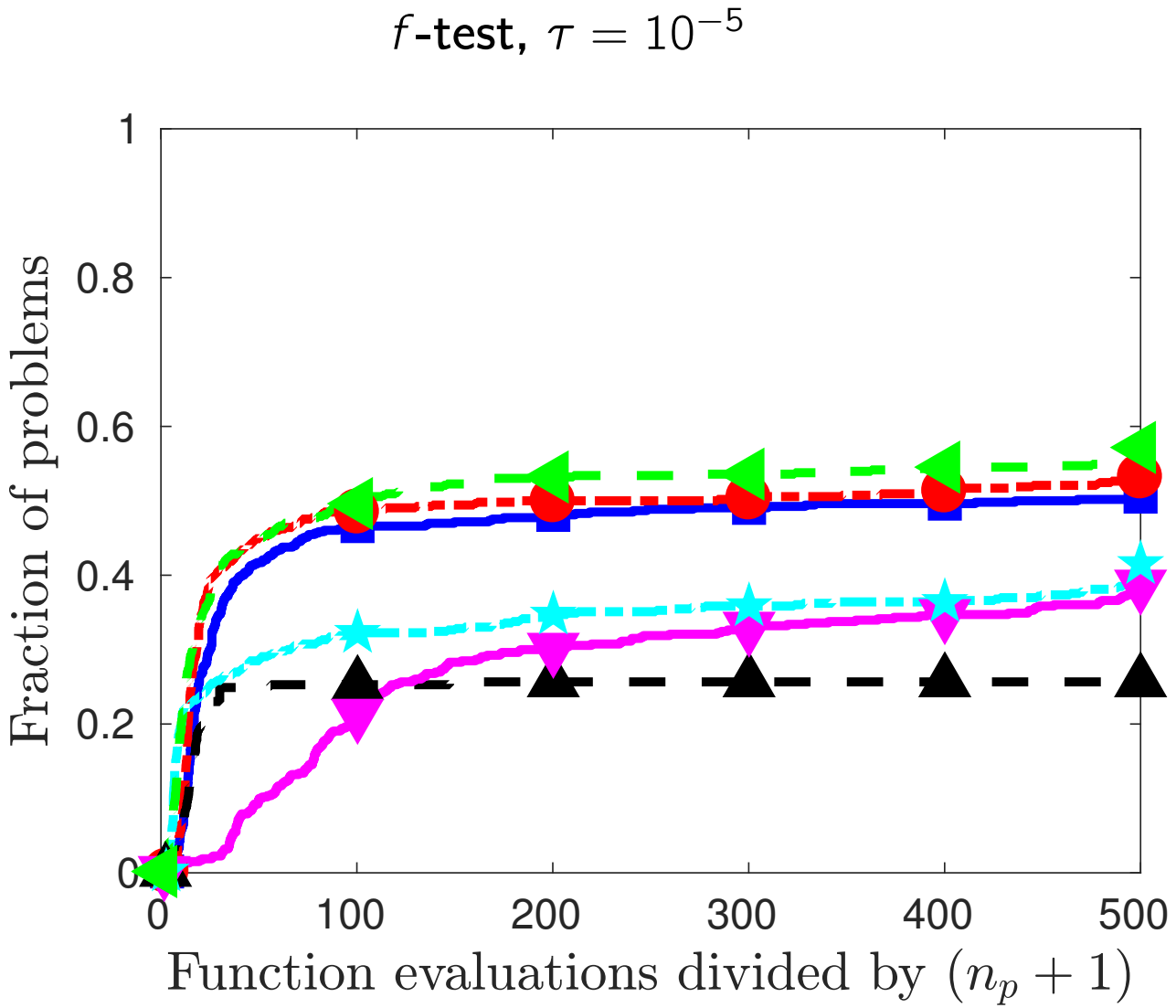

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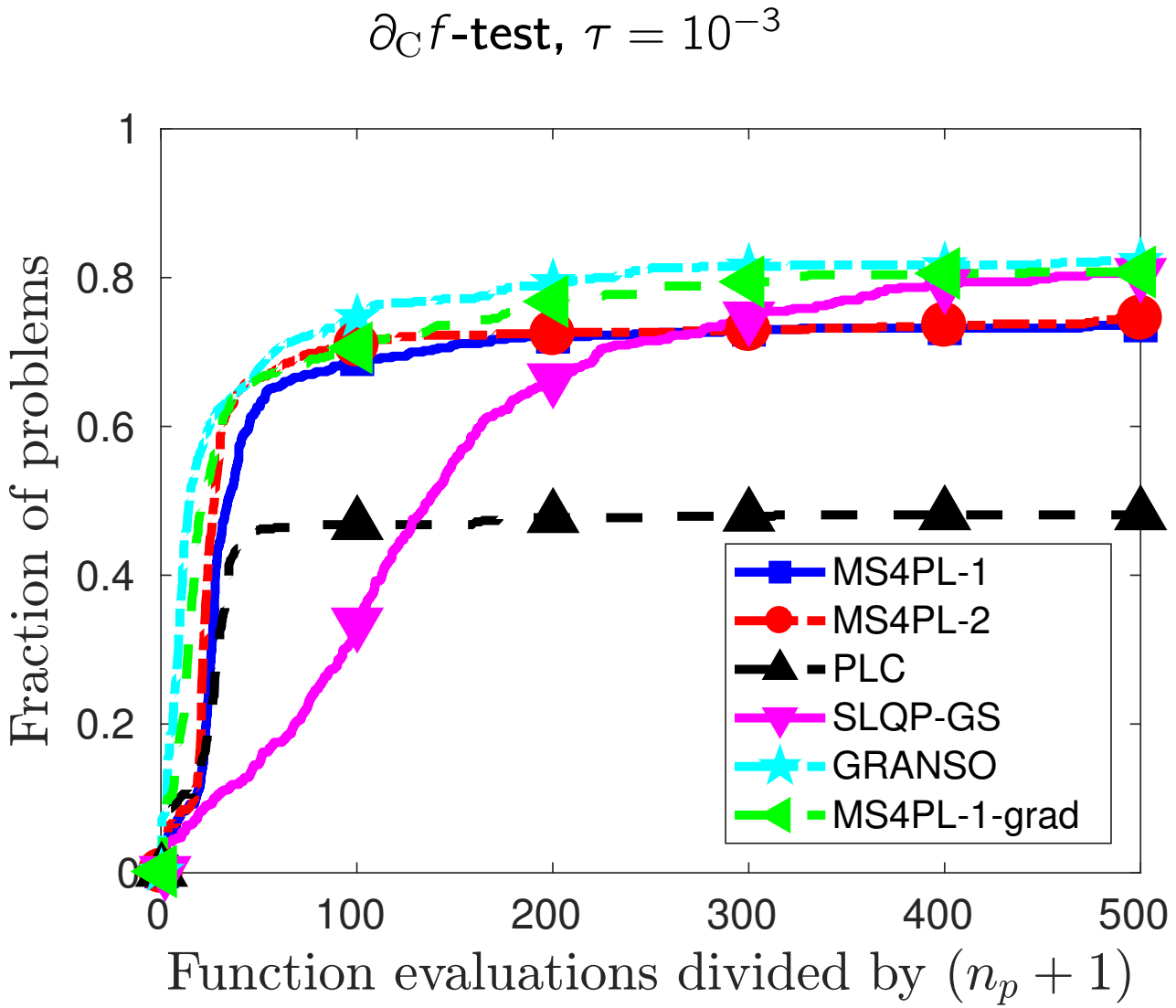

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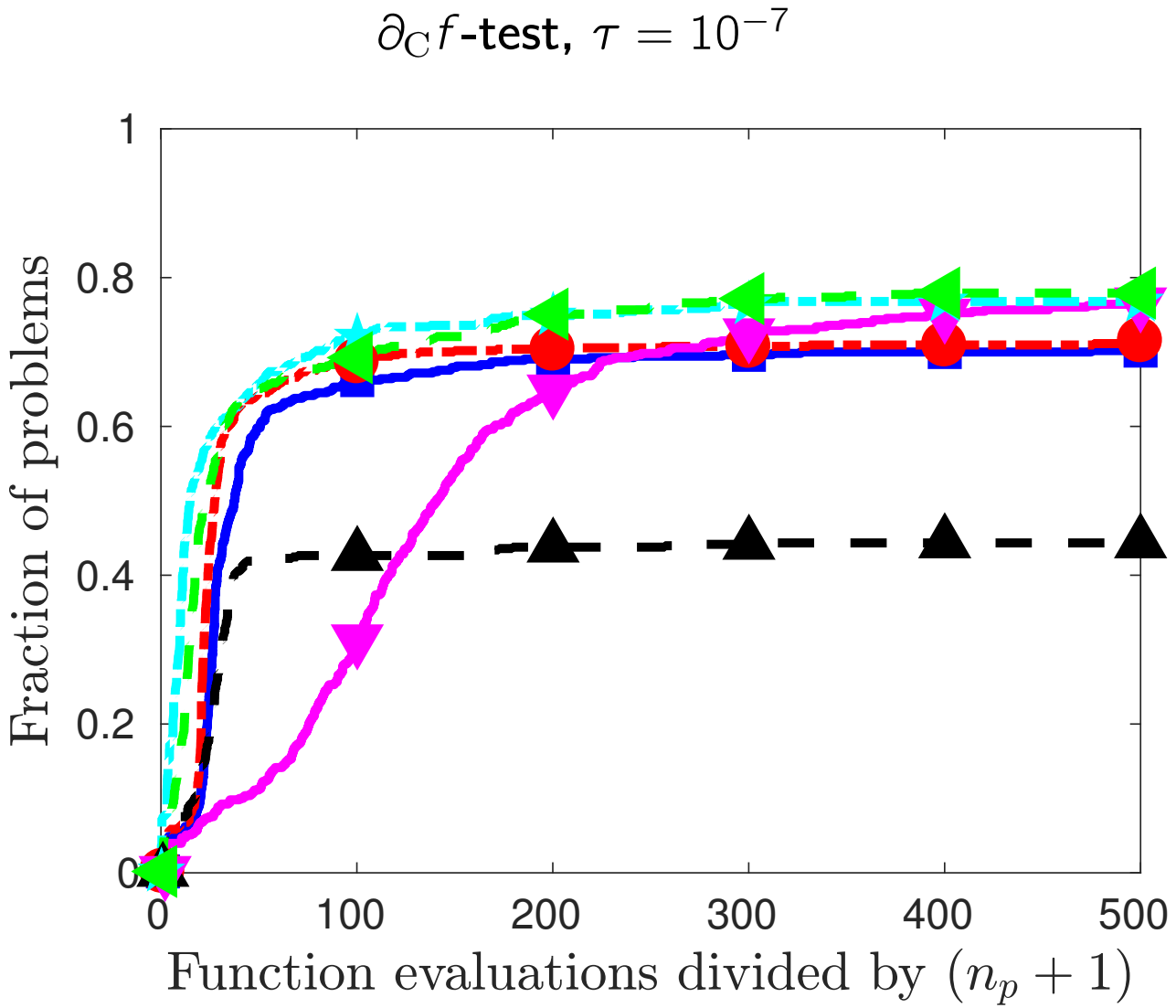

Conclusions

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Email `jmlarson@anl.gov` for a preprint.

Thank you!